LARGE-SCALE STRUCTURE BEYOND THE POWER SPECTRUM

Marcel Schmittfull

Berkeley Center for Cosmological Physics (BCCP)

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<u>In collaboration with</u>
Tobias Baldauf, Uros Seljak
Yu Feng, Florian Beutler, Blake Sherwin, Man Yat Chu

Berkeley BCCP Seminar, Sep 8 2015

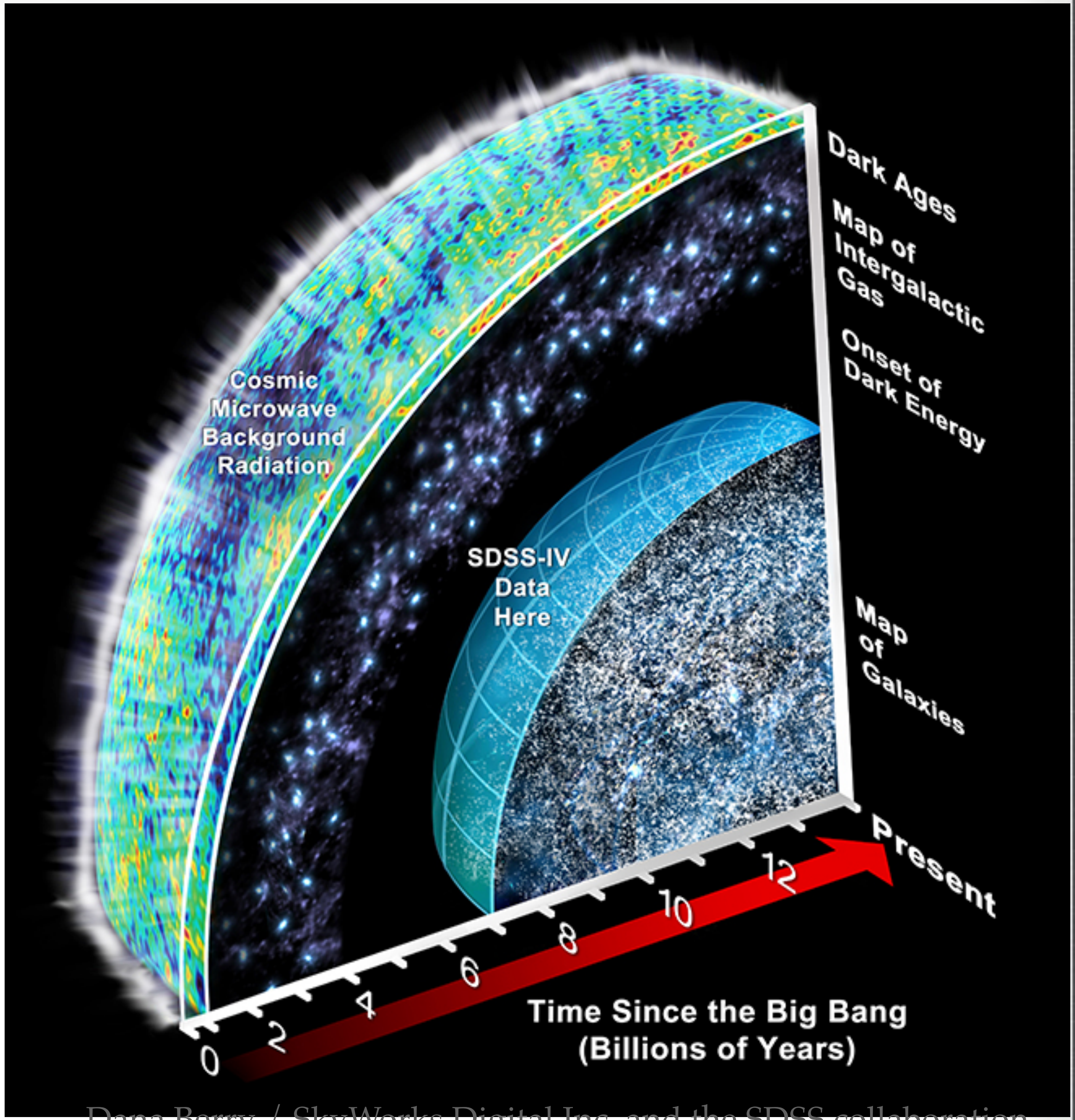
OVERVIEW

- Introduction
- Part I: Simple bispectrum estimators
- Part II: Eulerian reconstructions and N-point statistics

INTRODUCTION

MAIN COSMOLOGICAL PROBES

- Young universe (*380,000 yrs after big bang*)
  - [Gravity waves]
  - Cosmic Microwave Background (CMB)
- 'Recent' universe *(billions yrs after big bang)*
  - Large-scale structure (LSS)
    - 21cm, Lyα, CMB lensing, **galaxy clustering**, weak lensing
  - Supernovae
- All observations can be described with LCDM cosmological model
  (start with inflation, then expand with CDM and cosmological constant dark energy)

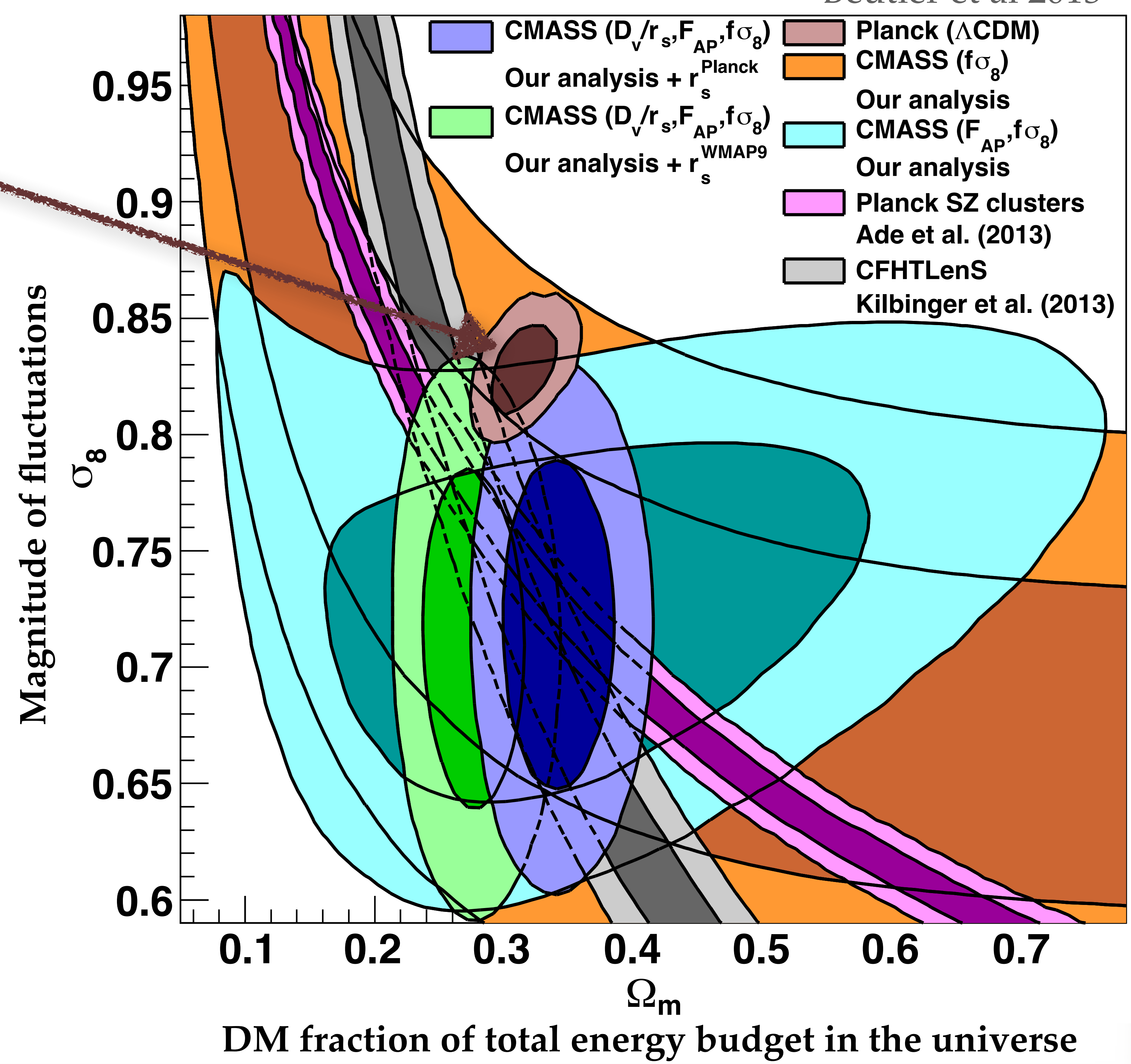


Dana Berry / SkyWorks Digital Inc. and the SDSS collaboration

MAIN COSMOLOGICAL PROBES

- Constraints on 6-parameter LCDM model:
  - CMB is most powerful

Beutler et al 2013



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- Constraints on 6-parameter LCDM model:
  - CMB is most powerful
- Free neutrino mass Σm_ν
  - Constraints degrade because Σm_ν and σ_8 degenerate in CMB
  - LSS helps

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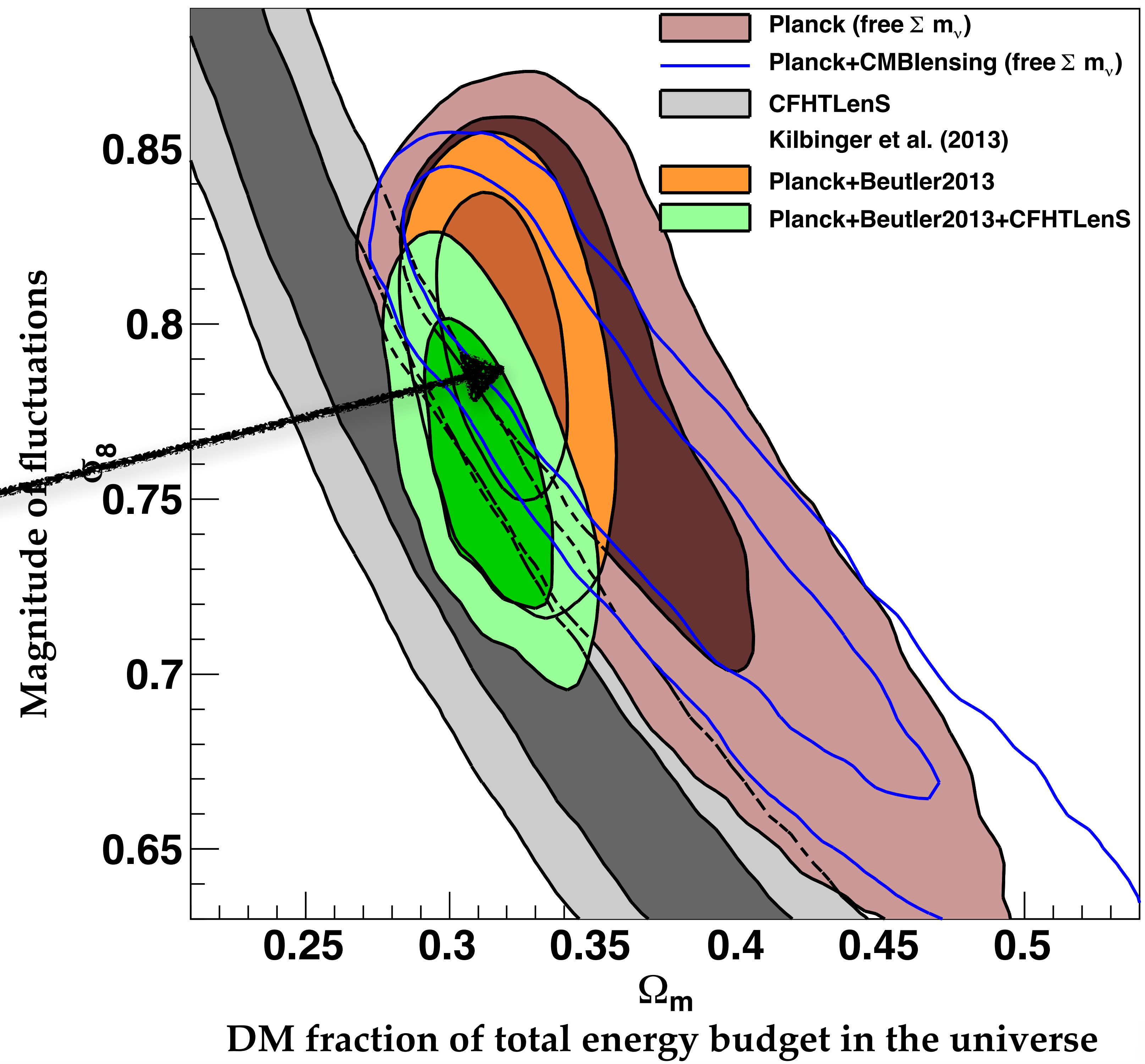

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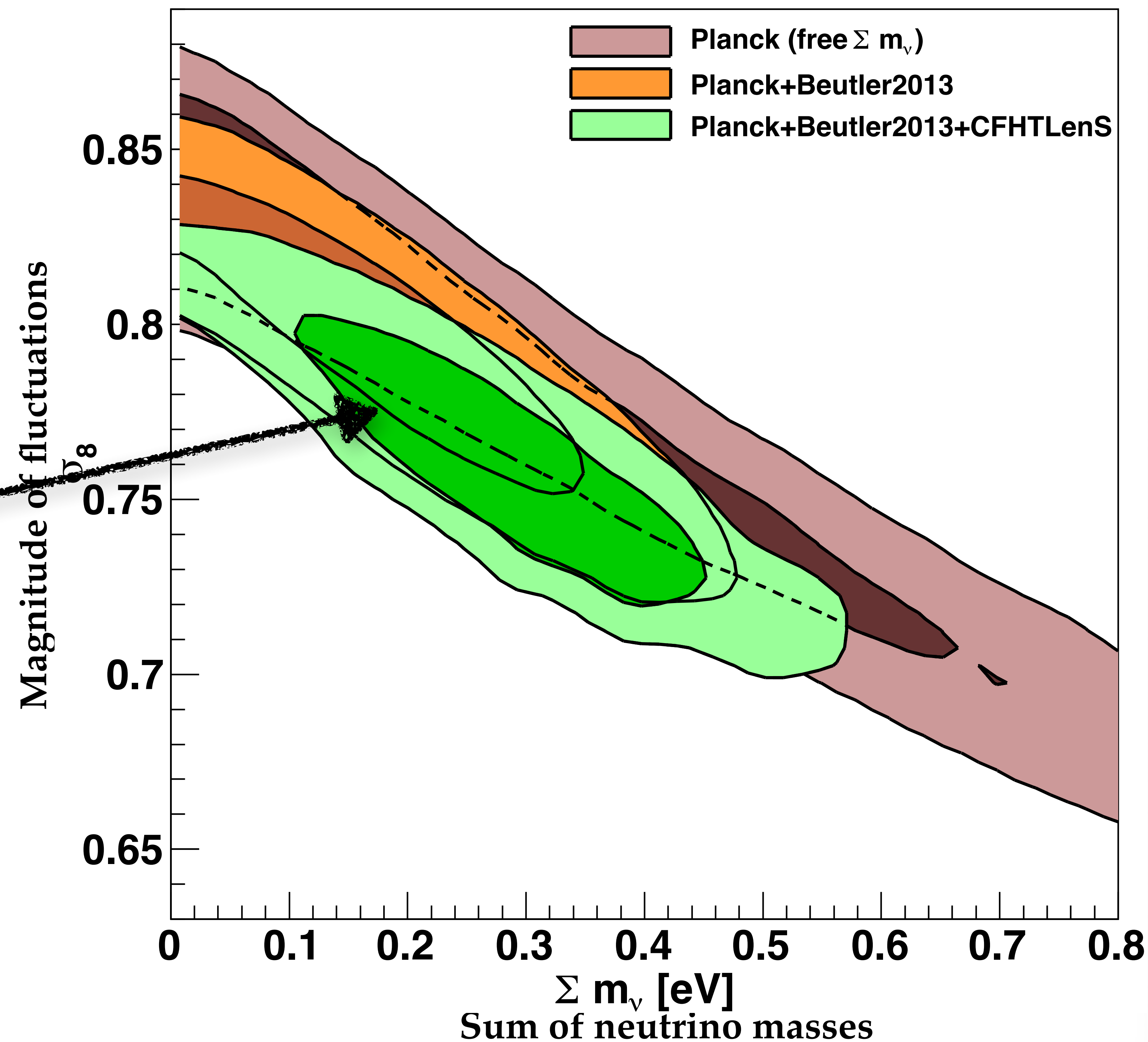

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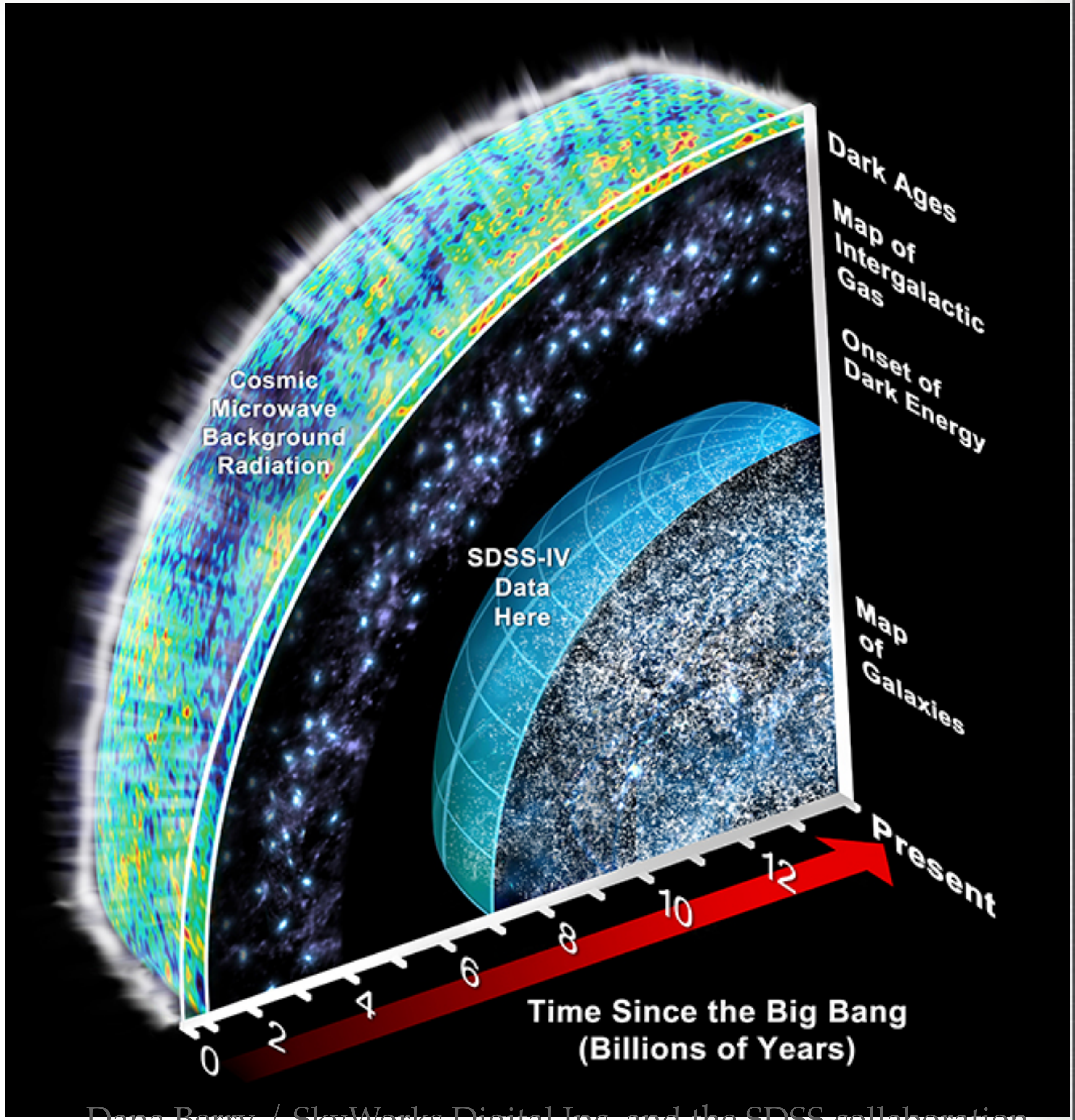
- Constraints on 6-parameter LCDM model:
  - CMB is most powerful
- Free neutrino mass Σm_ν
  - Constraints degrade because Σm_ν and σ_8 degenerate in CMB
  - LSS helpful
- Other model extensions (curvature, time-varying dark energy, mod. grav. etc)
  - LSS crucial



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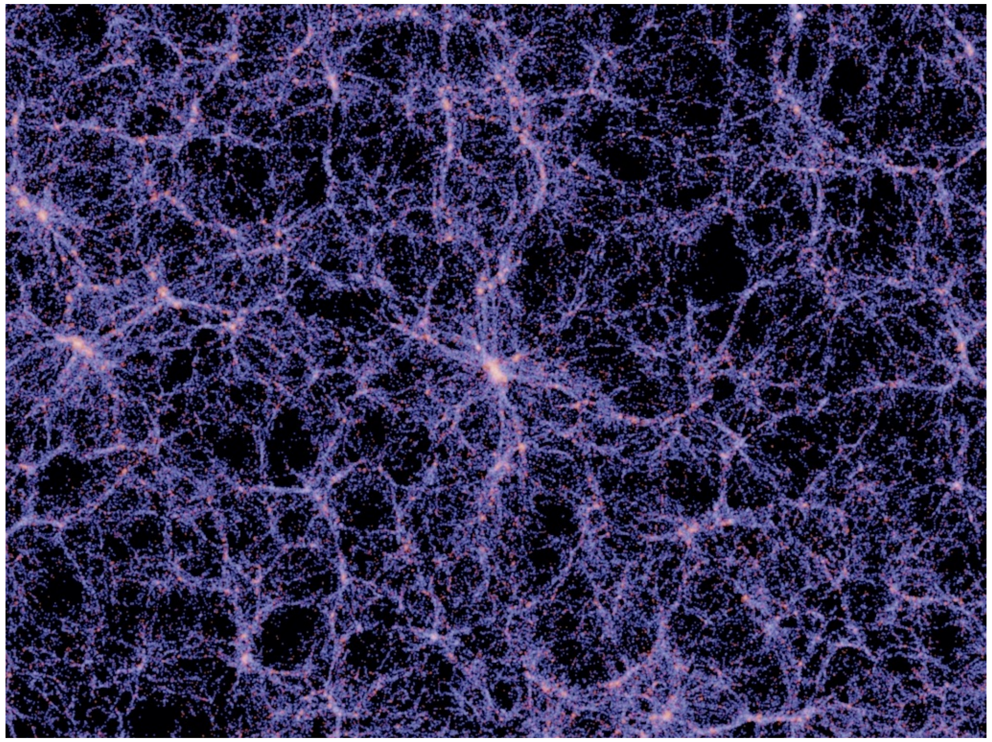
- Generally, should not just compare constraining power: LSS checks cosmological model as a whole (low z vs high z from CMB)

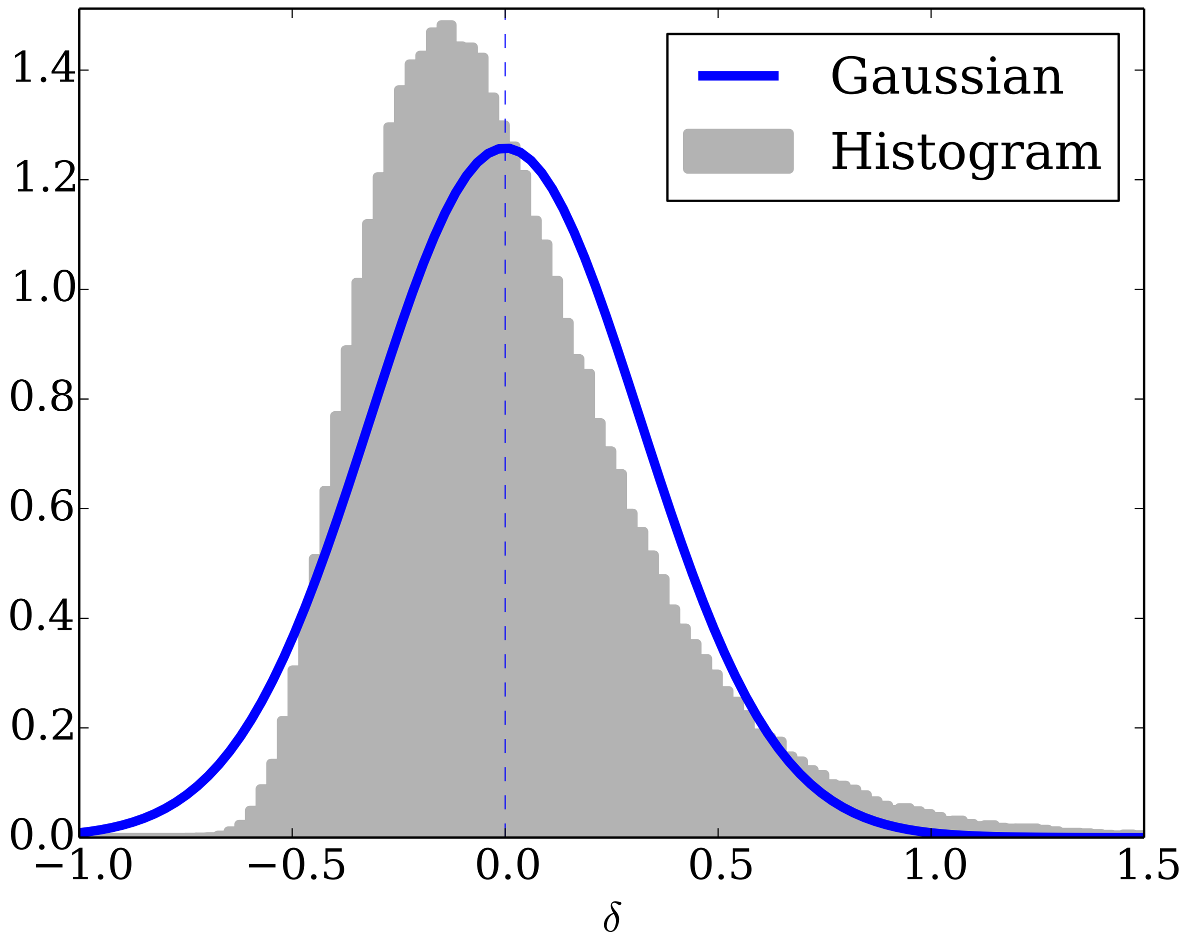
STATISTICAL PROPERTIES OF LSS

DARK MATTER DENSITY

- Equations of motion for **DM** are **non-linear**, containing products of Gaussian first-order perturbations (e.g. continuity equation $\partial_\eta \delta + \nabla[(1+\delta)\mathbf{v})] = 0$)

  ➟ DM density is not Gaussian (even in absence of primordial non-Gaussianity):





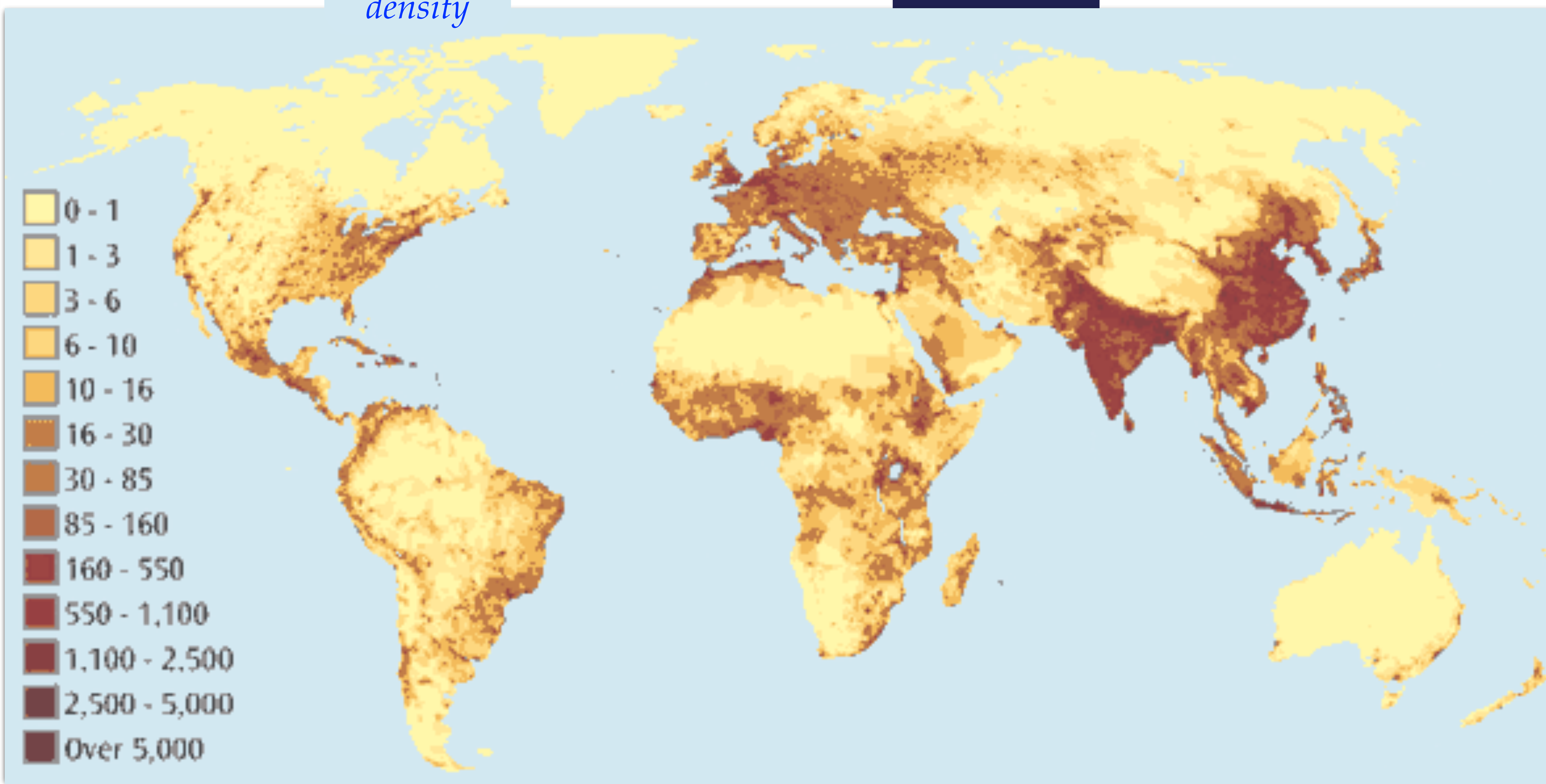
$k_{\max} \sim 0.15\ h/\mathrm{Mpc}$, $z = 0.55$, 2048^3 DM ptcles

NON-GAUSSIANITY FROM GRAVITY

- Another complication:
  - Distribution of dark matter is different from that of galaxies ➟ "Galaxy bias"

NON-GAUSSIANITY FROM GRAVITY

- Another complication:
  - Distribution of *human population density* is different from that of galaxies *night* ➟ "Galaxy bias"



Adopted from Tobias Baldauf

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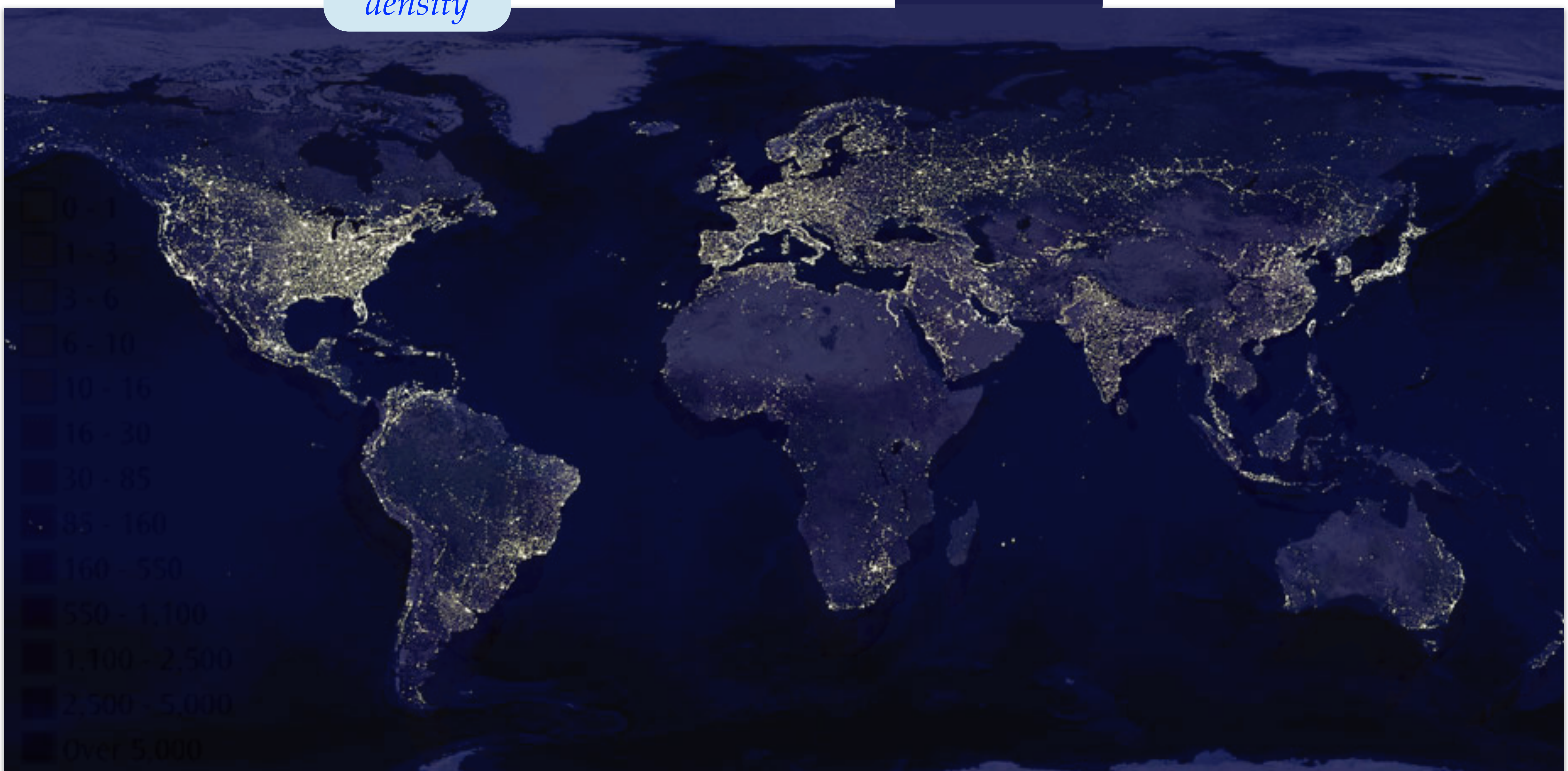
night



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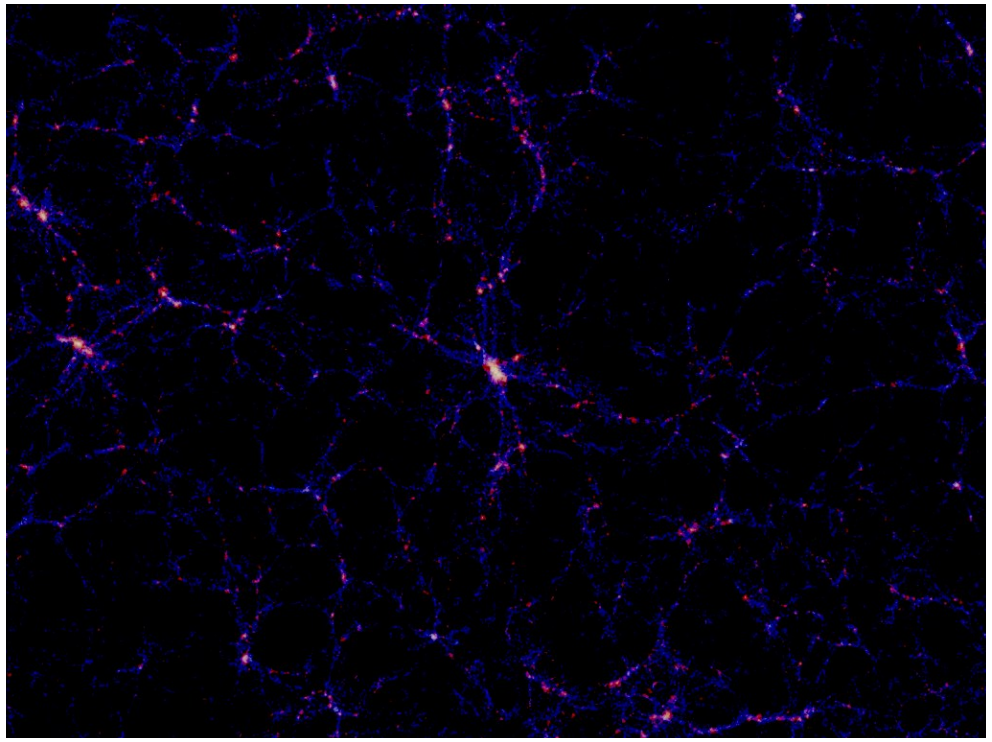
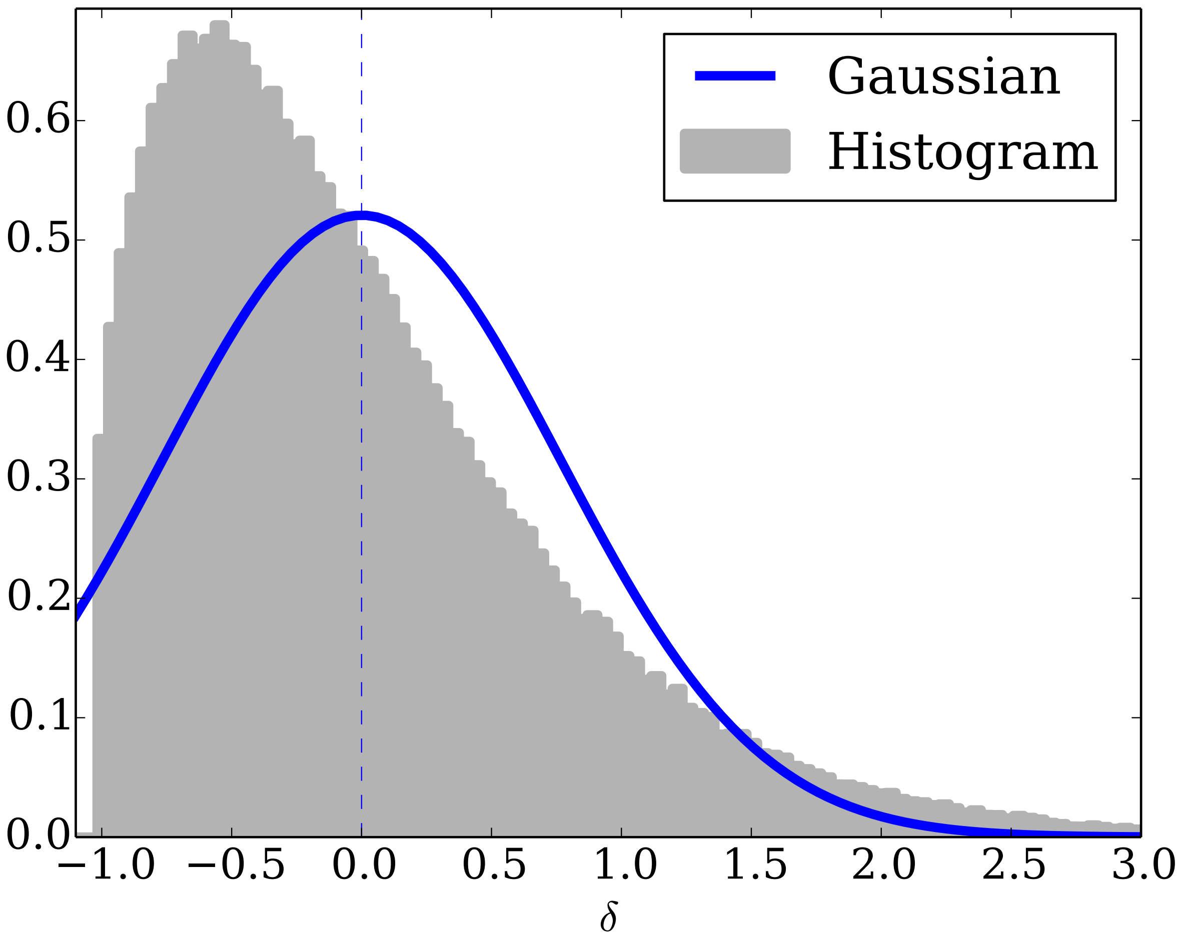
GALAXY DENSITY

- Galaxies are biased tracer of DM
- Bias relation has additional non-linearities $\delta_g(\mathbf{x}) \sim b_1 \delta_m(\mathbf{x}) + b_2 \delta_m^2(\mathbf{x}) + b_{s^2} s_m^2(\mathbf{x})$

  ➟ pdf of galaxy density is even more non-Gaussian than that for DM:





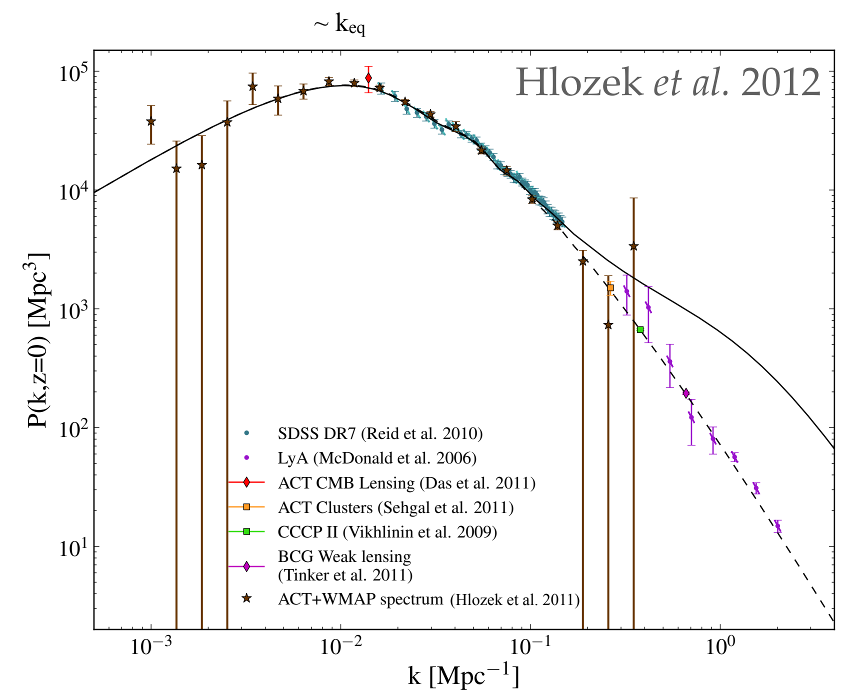
$k_{\max} \sim 0.15\ h/\mathrm{Mpc}$, $z = 0.55$, halos with $b_1 \sim 2$ and mass 1-4 x $10^{13}\ M_{\mathrm{solar}}/h$

QUANTIFYING THE PDF

POWER AND BISPECTRUM

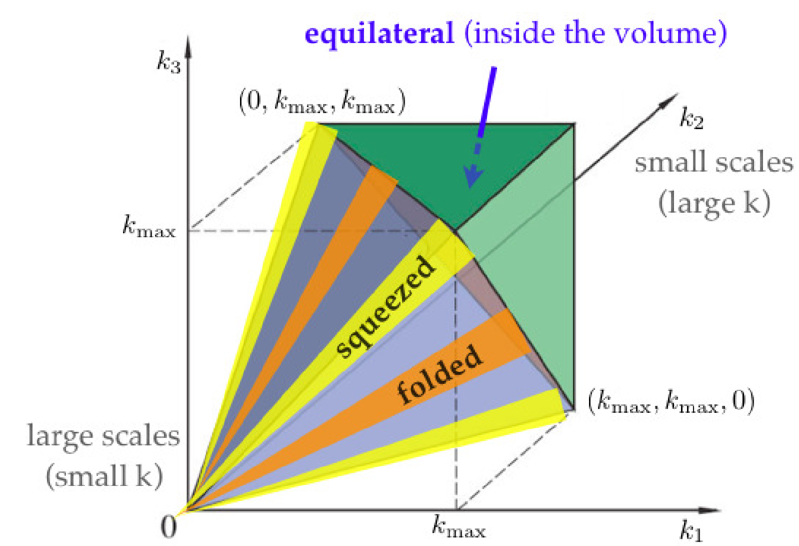
- **Gaussian** field is completely specified by its **power spectrum P_δ**
  (2-point correlation function in Fourier space)

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2) P_\delta(k_1)$$



- Primary diagnostic for **non-Gaussianity** is the **bispectrum B_δ**
  (3-point correlation function in Fourier space: given 2 over-densities, specifies probability of having a third overdensity)

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle$$
$$= (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\delta(k_1, k_2, k_3)$$



bispectrum drawn on space of triangle configurations

MOTIVATION FOR MEASURING BISPECTRUM

- ***Late universe***: Break parameter degeneracies, e.g. between b_1 and σ_{8}:
  - Power spectrum: $P_{\mathrm{hh}} \propto b_1^2 \sigma_8^2$
  - Bispectrum: $B_{\mathrm{hhh}} \propto\sim b_1^3 \sigma_8^4 + \sim b_1^2 b_2 \sigma_8^4 + \cdots$
  - Lots of upcoming LSS experiments that could benefit, e.g. eBOSS, DESI, EUCLID, WFIRST, LSST, …

Fry 1994
Verde *et al.* 1997-2002
Scoccimarro *et al.* 1998
Sefusatti *et al.* 2006
Gil-Marin *et al.* 2014

- ***Early universe***: Constrain primordial non-Gaussianity
  - CMB close to cosmic variance limit, so need LSS
  - $f_{\mathrm{NL}}^{\mathrm{loc}}$: Bispectrum complementary to $P(k)$ and less affected by low-k systematics
  - $f_{\mathrm{NL}}^{\mathrm{eq}}$: No signal in $P(k)$ → need bispectrum
  - e.g. SPHEREX (proposed satellite)

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But: Beyond 2-point plagued by **increased complexity** of analysis (covariances, window function, fiber collisions, computational cost ~ N^6; non-linear DM, bias, redshift-space distortions, galaxy-halo connection, …)

SIMPLE BISPECTRUM ESTIMATORS

THEORY **DM** BISPECTRUM

- Non-linear DM density (2nd order SPT)

$\delta_{\rm m}$: *nonlinear* DM density
δ_0: linear DM density
$\mathbf{\Psi}_0$: linear displacement $\equiv -\frac{i\mathbf{k}}{k^2}\delta_0(\mathbf{k})$
s_0: linear tidal tensor

$$\delta_{\rm m}(\mathbf{x}) = \underbrace{\delta_0(\mathbf{x})}_{\text{linear}} + \underbrace{\frac{17}{21}\delta_0^2(\mathbf{x})}_{\text{nonlinear growth}} + \underbrace{\mathbf{\Psi}_0(\mathbf{x})\cdot\nabla\delta_0(\mathbf{x})}_{\text{shift}} + \underbrace{\frac{4}{21}s_0^2(\mathbf{x})}_{\text{tidal}}$$

$$\Rightarrow \quad \langle\delta_{\rm m}(\mathbf{k}_1)\delta_{\rm m}(\mathbf{k}_2)\delta_{\rm m}(\mathbf{k}_3)\rangle \sim 2P^{\rm lin}_{\rm mm}(k_1)P^{\rm lin}_{\rm mm}(k_2)F_2(\mathbf{k}_1,\mathbf{k}_2) + \text{ 2 perms in } k_1,k_2,k_3$$

where

$$F_2(\mathbf{k}_1,\mathbf{k}_2) = \underbrace{\frac{17}{21}}_{\text{nonlinear growth}} + \underbrace{\frac{1}{2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right)\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2}_{\text{shift}} + \underbrace{\frac{4}{21}\frac{3}{2}\left((\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2)^2-\frac{1}{3}\right)}_{\text{tidal}}$$

THEORY **HALO** BISPECTRUM

δ_{m}: *nonlinear* DM density
δ_{h}: halo density
s_{m}: DM tidal tensor

- Non-linear halo bias

$$\delta_{\mathrm{h}}(\mathbf{x}) = b_1 \delta_{\mathrm{m}}(\mathbf{x}) + b_2 \left[\delta_{\mathrm{m}}^2(\mathbf{x}) - \langle \delta_{\mathrm{m}}^2(\mathbf{x}) \rangle\right] + \frac{2}{3} b_{s^2} \left[s_{\mathrm{m}}^2(\mathbf{x}) - \langle s_{\mathrm{m}}^2(\mathbf{x}) \rangle\right]$$

$$\Rightarrow \langle \delta_{\mathrm{h}}(\mathbf{k}_1)\delta_{\mathrm{h}}(\mathbf{k}_2)\delta_{\mathrm{h}}(\mathbf{k}_3)\rangle$$

$$\sim 2P_{\mathrm{mm}}^{\mathrm{lin}}(k_1)P_{\mathrm{mm}}^{\mathrm{lin}}(k_2)\left[b_1^3 F_2(\mathbf{k}_1, \mathbf{k}_2) + b_1^2 b_2 + \frac{2}{3} b_1^2 b_{s^2} \mathsf{P}_2(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2)\right] + 2 \text{ perms}$$

THEORY HALO BISPECTRUM

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δ_{h}: halo density
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- Non-linear halo bias

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- Decompose in Legendre polys P_l with $l = 0, 1, 2$

$$B^{(l=0)}_{\mathrm{hhh}} = \left(\frac{34}{21} b_1^3 + 2b_1^2 b_2\right) P^{\mathrm{lin}}_{\mathrm{mm}}(k_1)P^{\mathrm{lin}}_{\mathrm{mm}}(k_2) + 2 \text{ perms},$$

$$B^{(l=1)}_{\mathrm{hhh}} = b_1^3 \left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right) P^{\mathrm{lin}}_{\mathrm{mm}}(k_1)P^{\mathrm{lin}}_{\mathrm{mm}}(k_2)\mathsf{P}_1(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2) + 2 \text{ perms},$$

$$B^{(l=2)}_{\mathrm{hhh}} = \left(\frac{8}{21} b_1^3 + \frac{4}{3} b_1^2 b_{s^2}\right) P^{\mathrm{lin}}_{\mathrm{mm}}(k_1)P^{\mathrm{lin}}_{\mathrm{mm}}(k_2)\mathsf{P}_2(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2) + 2 \text{ perms}.$$

BISPECTRUM ESTIMATION

- ***Goal:*** Given DM/halo density δ, estimate *coefficients* of all bispectrum contributions
  - ➟ these depend on bias and cosmological parameters that we aim to extract

- ***Method:*** Maximum likelihood estimators for the coefficient of contribution B^{contri}

$$\hat{\mathrm{coeff}}(B^{\mathrm{contri}}|\delta) \propto \int_{\mathbf{k},\mathbf{q}} \underbrace{B^{\mathrm{contri}}(\mathbf{q},\mathbf{k}-\mathbf{q},-\mathbf{k})}_{\text{theory template}} \underbrace{\frac{\delta(\mathbf{q})\delta(\mathbf{k}-\mathbf{q})\delta(-\mathbf{k})}{P_\delta(q)P_\delta(|\mathbf{k}-\mathbf{q}|)P_\delta(k)}}_{\text{inv.}-\text{variance weighted data}}$$

- ***Example:*** $B^{\mathrm{contri}} = P(k_1)P(k_2)$

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$$\Rightarrow \quad \hat{\mathrm{coeff}} \propto \int \mathrm{d}k \frac{k^2}{P(k)} \hat{P}_{\delta^2,\delta}(k)$$

relies on separability of LSS bispectrum

$$\downarrow$$

$$\hat{P}_{\delta^2,\delta}(k) \sim \sum_{\mathbf{k},|\mathbf{k}|=k} [\delta^2](\mathbf{k})\delta(-\mathbf{k})$$

= $'\langle\delta^2|\delta\rangle'$ = cross-spectrum of $\delta^2(\mathbf{x})$ and δ

GET *ALL* BISPECTRUM CONTRIS FROM 3 CROSS-SPECTRA

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l=0 growth

squared density × density

$$\hat{P}_{\delta^2,\delta}(k) \sim \sum_{\mathbf{k},|\mathbf{k}|=k} [\delta^2](\mathbf{k})\ \delta(-\mathbf{k})$$

- nonlinear DM growth δ^2
- quadratic bias b_2

l=1 shift

displacement dot density gradient × density

$$\hat{P}_{-\Psi^i\partial_i\delta,\delta}(k) \sim \sum_{\mathbf{k},|\mathbf{k}|=k} [-\Psi^i\partial_i\delta](\mathbf{k})\ \delta(-\mathbf{k})$$

- nonlinear DM shift $\boldsymbol{\Psi}(\mathbf{x})\cdot\nabla\delta(\mathbf{x})$

$$\Psi^i(\mathbf{k}) \equiv -\frac{ik^i}{k^2}\delta(\mathbf{k})$$

l=2 tidal

tidal tensor × density

$$\hat{P}_{s^2,\delta}(k) \sim \sum_{\mathbf{k},|\mathbf{k}|=k} [s^2](\mathbf{k})\ \delta(-\mathbf{k})$$

- nonlinear DM tidal term
- tidal tensor bias b_{s^2}

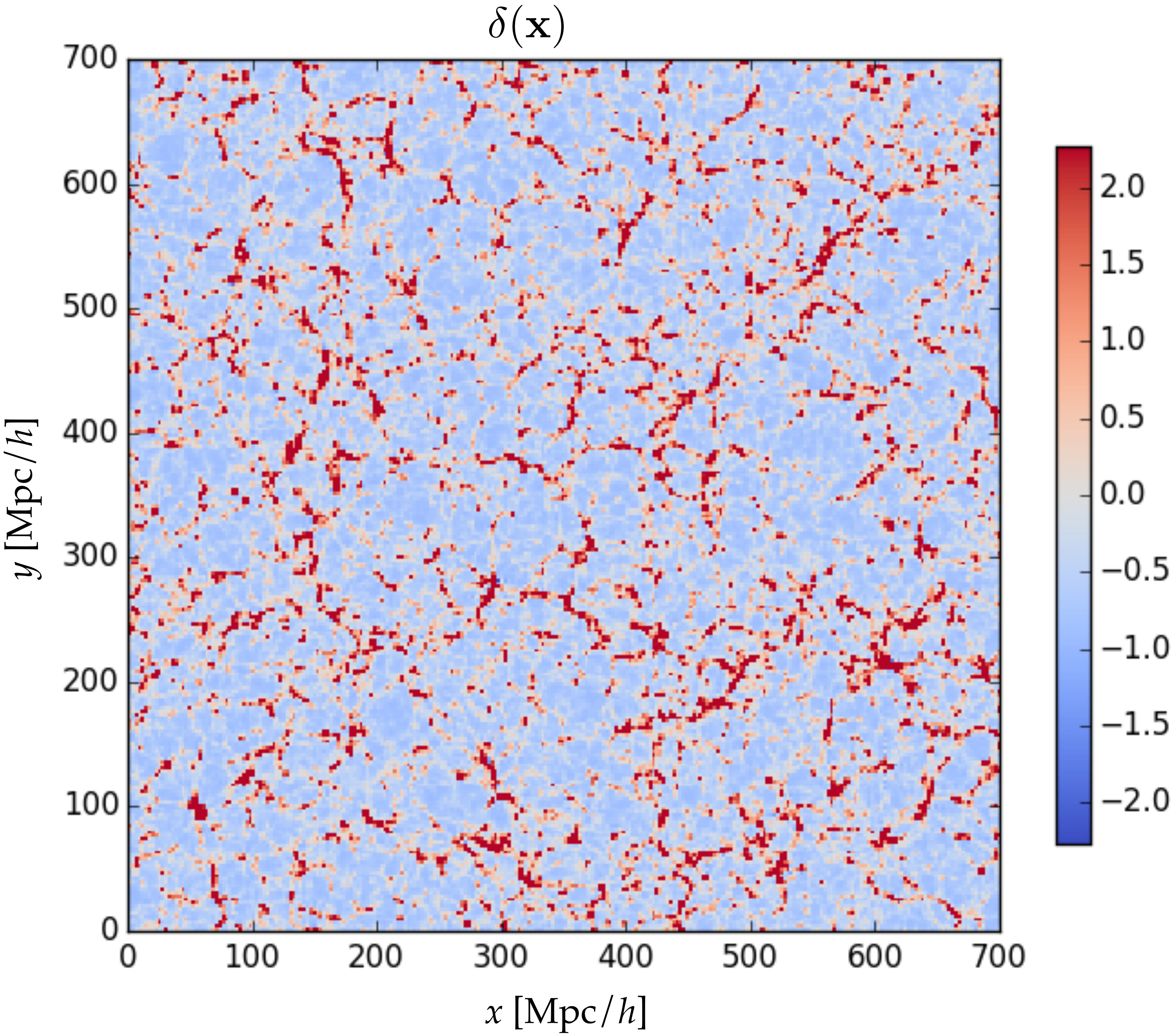
$$s^2(\mathbf{x}) \equiv \frac{3}{2}s_{ij}(\mathbf{x})s_{ij}(\mathbf{x})$$

$$s_{ij}(\mathbf{k}) = \left(\frac{k_ik_j}{k^2} - \frac{1}{3}\delta^{(\mathrm{K})}_{ij}\right)\delta(\mathbf{k})$$

IN PICTURES

MS, Baldauf, Seljak, 1411.6595

- *1. Start with density*



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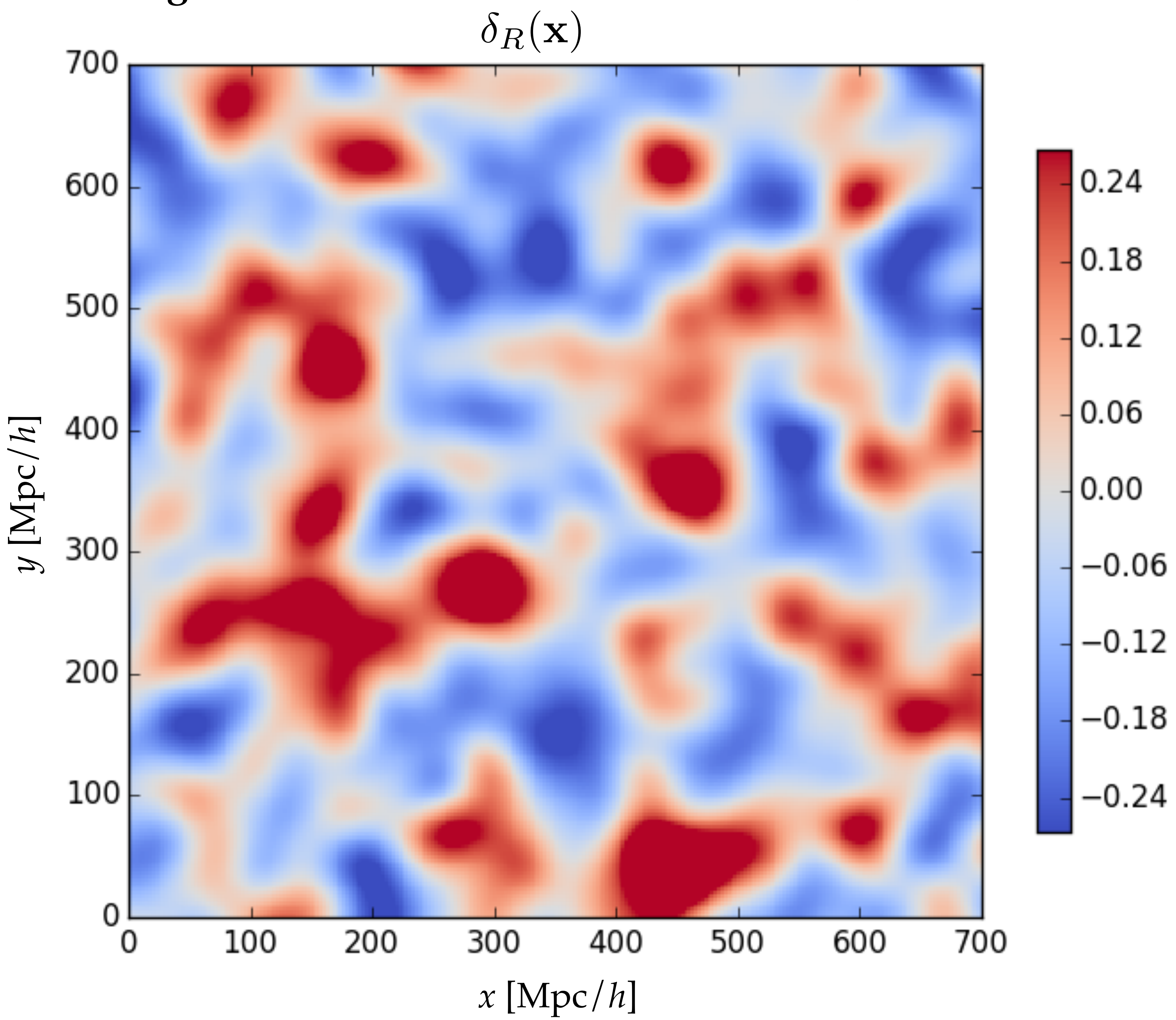
- *2. Apply smoothing* (here Gaussian with R=20 Mpc/h)



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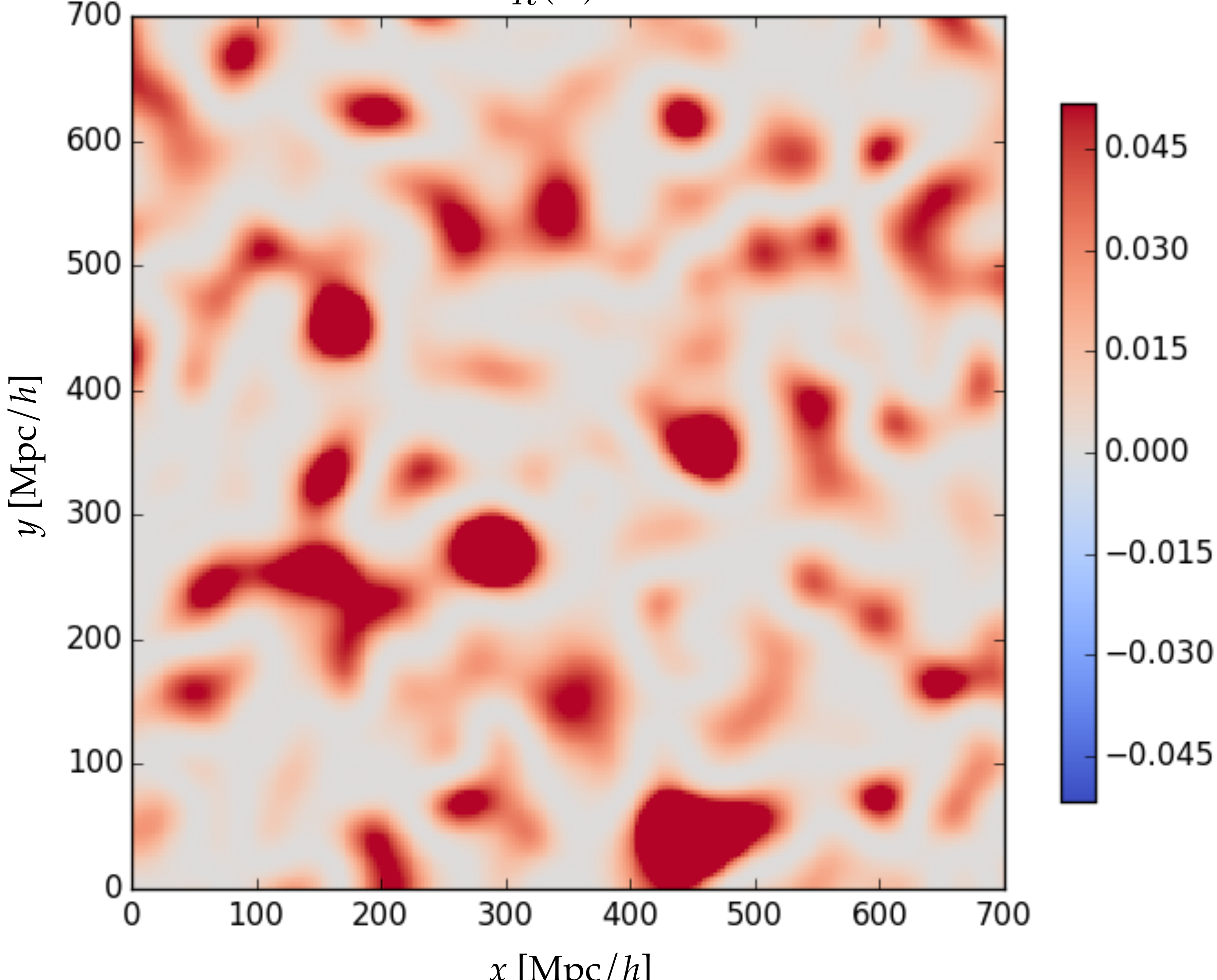
- ***3. Get squared density***

$\delta_R^2(\mathbf{x})$



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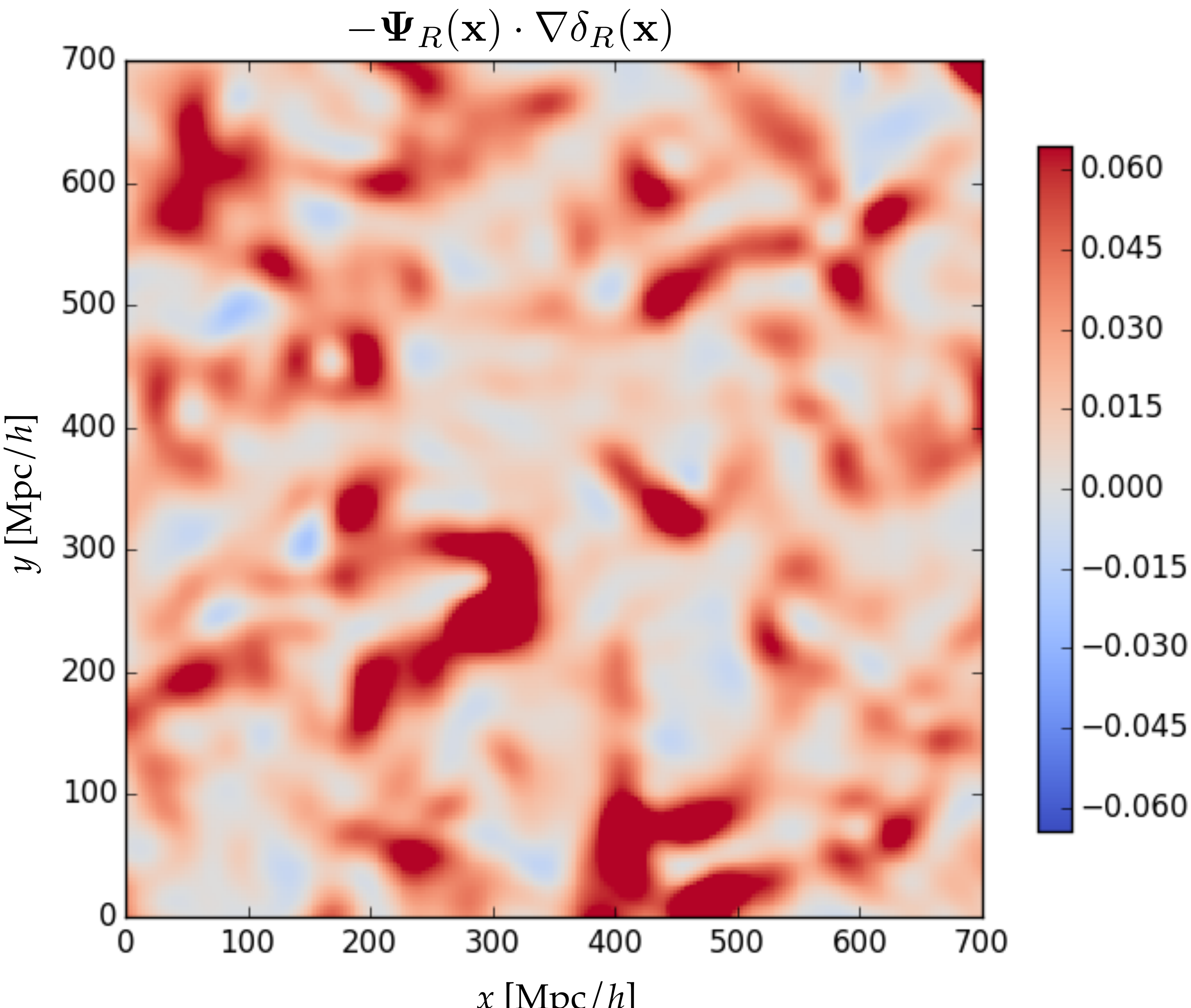
- 4. *Get shift term*



IN PICTURES

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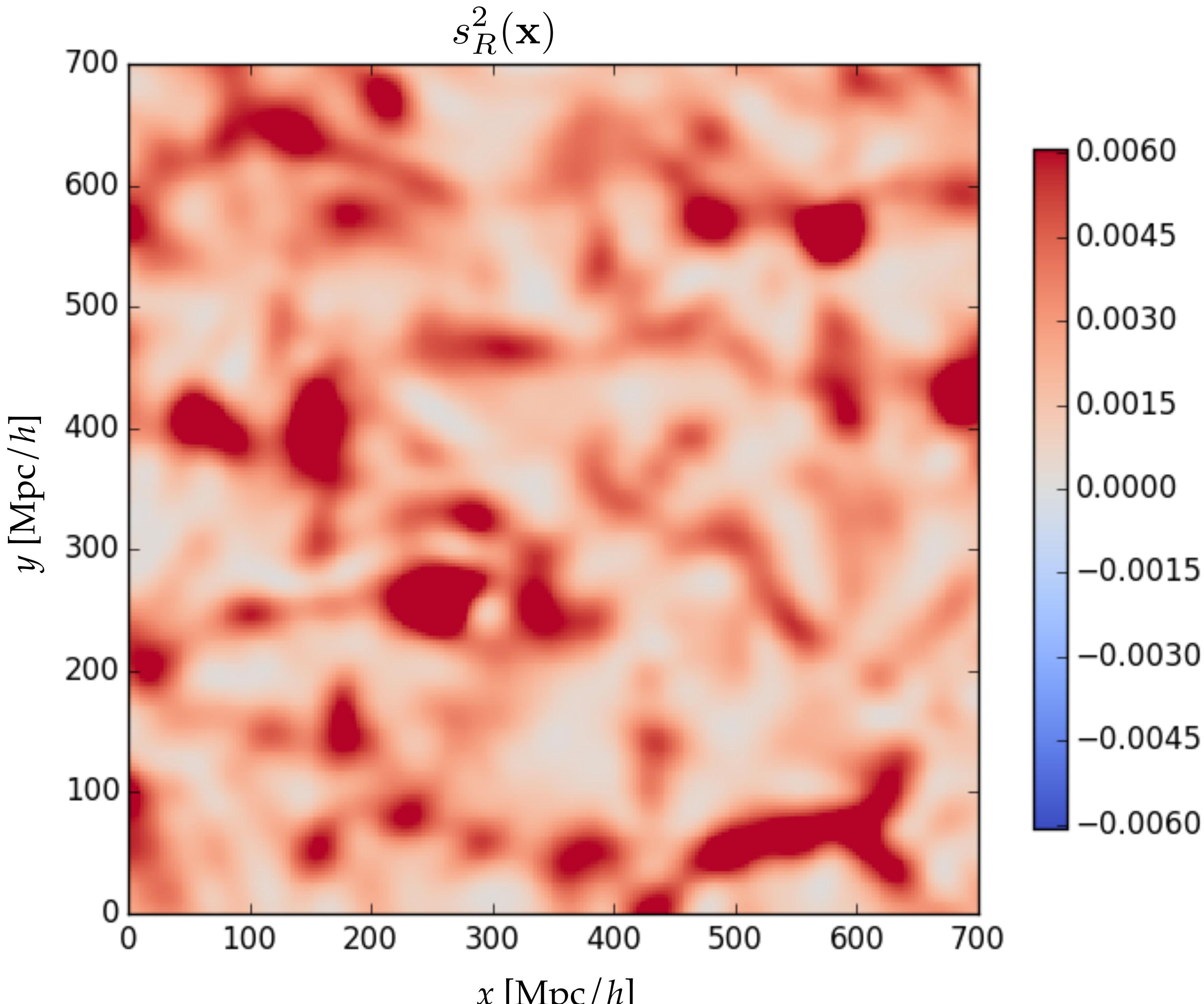
- 5. *Get tidal term*



IN PICTURES

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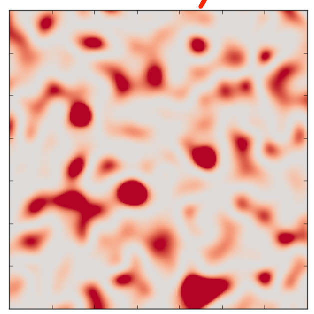
▶ 6. *Cross-correlate*

***l*=0 growth**

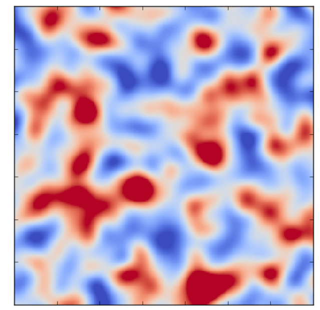
$\hat{P}_{\delta^2,\delta}(k) \sim$

squared density



X

density



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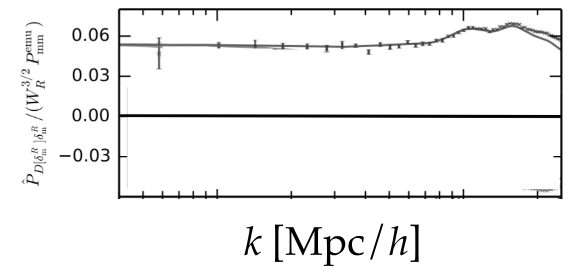
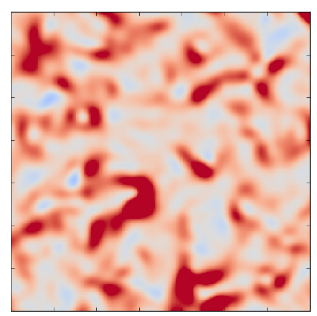


- nonlinear DM growth δ^2
- quadratic bias b_2

***l*=1 shift**

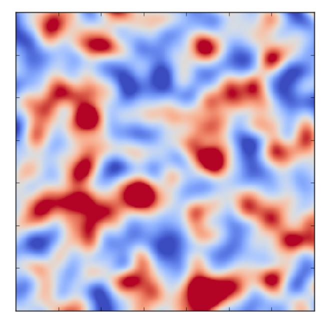
$\hat{P}_{-\Psi^i\partial_i\delta,\delta}(k) \sim$

displacement dot density gradient



X

density



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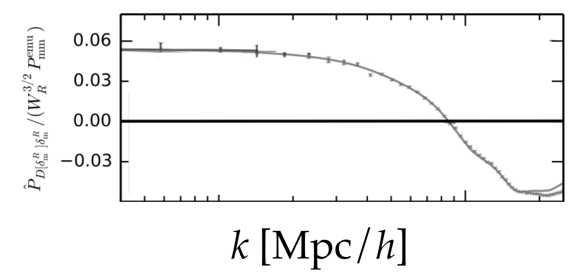
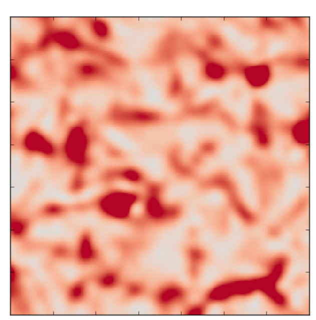


- nonlinear DM shift $\mathbf{\Psi}(\mathbf{x}) \cdot \nabla\delta(\mathbf{x})$

***l*=2 tidal**

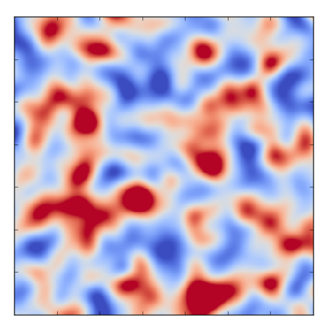
$\hat{P}_{s^2,\delta}(k) \sim$

tidal tensor



X

density



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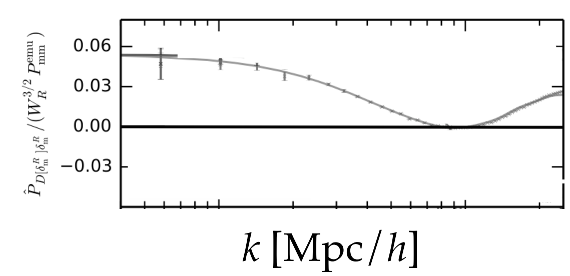

- nonlinear DM tidal term
- tidal tensor bias

PRIMORDIAL NON-GAUSSIANITY

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- Optimal $f_{\rm NL}^{\rm loc}$ estimator from DM bispectrum:

$$\hat{f}_{\rm NL}^{\rm loc} = \frac{24\pi L^3}{N_{\rm loc}} \int \mathrm{d}k \frac{k^2 M^2(k)}{P_{\rm mm}(k)} \hat{P}_{[\frac{\delta_{\rm m}}{M}]^2, \frac{\delta_{\rm m}}{M}}(k)$$

$$M(k,z) \equiv \frac{2}{3} \frac{k^2 T(k) D(z)}{\Omega_{\rm m} H_0^2}$$ (Poisson factor)

➟ basically just ϕ^2 cross ϕ, where $\phi \sim \delta/k^2$

➟ in principle same S/N as getting $f_{\rm NL}^{\rm loc}$ from measurement of the entire LSS bispectrum

THEORETICAL MODELING

MODELING CROSS-SPECTRA

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- Expectation value of cross-spectrum with *general* kernel $D \in \{\mathsf{P}_0, -F_2^1\mathsf{P}_1, \mathsf{P}_2\}$

$$P_{D[\delta_a^R],\delta_b^R}(k) = W_R(k)\int\frac{\mathrm{d}^3q}{(2\pi)^3}W_R(q)W_R(|\mathbf{k}-\mathbf{q}|)D(\mathbf{q},\mathbf{k}-\mathbf{q})B_{\delta_a\delta_a\delta_b}(\mathbf{q},\mathbf{k}-\mathbf{q},-\mathbf{k})$$

integrated bispectrum

MODELING CROSS-SPECTRA

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integrated bispectrum

- Leading-order SPT for *hhh* cross-spectra

$$\begin{aligned}P_{D[\delta_\mathrm{h}^R],\delta_\mathrm{h}^R}(k) = &\left(\frac{34}{21}b_1^3 + 2b_1^2 b_2\right)\left[I^R_{D\mathsf{P}_0}(k) + 2I^{\mathrm{bare},R}_{D\mathsf{P}_0}(k)\right] \\ &+ 2b_1^3\left[I^R_{D,F_2^1\mathsf{P}_1}(k) + 2I^{\mathrm{bare},R}_{D,F_2^1\mathsf{P}_1}(k)\right] \\ &+ \left(\frac{8}{21}b_1^3 + \frac{4}{3}b_1^2 b_{s^2}\right)\left[I^R_{D\mathsf{P}_2}(k) + 2I^{\mathrm{bare},R}_{D\mathsf{P}_2}(k)\right]\end{aligned}$$

$$I^R_{DE}(k) \equiv W_R(k)\int \frac{\mathrm{d}^3q}{(2\pi)^3} W_R(q)W_R(|\mathbf{k}-\mathbf{q}|)P^{\mathrm{lin}}_{\mathrm{mm}}(q)P^{\mathrm{lin}}_{\mathrm{mm}}(|\mathbf{k}-\mathbf{q}|)D(\mathbf{q},\mathbf{k}-\mathbf{q})E(\mathbf{q},\mathbf{k}-\mathbf{q})$$

$$I^{\mathrm{bare},R}_{DE}(k) \equiv W_R(k)P^{\mathrm{lin}}_{\mathrm{mm}}(k)\int \frac{\mathrm{d}^3q}{(2\pi)^3} W_R(q)W_R(|\mathbf{k}-\mathbf{q}|)P^{\mathrm{lin}}_{\mathrm{mm}}(q)D(\mathbf{q},\mathbf{k}-\mathbf{q})E(\mathbf{q},-\mathbf{k})$$

MODELING CROSS-SPECTRA

MS, Baldauf, Seljak, 1411.6595

- Expectation value of cross-spectrum with *general* kernel $D \in \{\mathsf{P}_0, -F_2^1\mathsf{P}_1, \mathsf{P}_2\}$

$$P_{D[\delta_a^R],\delta_b^R}(k) = W_R(k)\int\frac{\mathrm{d}^3q}{(2\pi)^3}W_R(q)W_R(|\mathbf{k}-\mathbf{q}|)D(\mathbf{q},\mathbf{k}-\mathbf{q})B_{\delta_a\delta_a\delta_b}(\mathbf{q},\mathbf{k}-\mathbf{q},-\mathbf{k})$$

integrated bispectrum

- Leading-order SPT for *hhh* cross-spectra

$$\begin{aligned}P_{D[\delta_\mathrm{h}^R],\delta_\mathrm{h}^R}(k) =& \left(\frac{34}{21}b_1^3 + 2b_1^2b_2\right)\left[I^R_{D\mathsf{P}_0}(k) + 2I^{\mathrm{bare},R}_{D\mathsf{P}_0}(k)\right]\\ &+ 2b_1^3\left[I^R_{D,F_2^1\mathsf{P}_1}(k) + 2I^{\mathrm{bare},R}_{D,F_2^1\mathsf{P}_1}(k)\right]\\ &+ \left(\frac{8}{21}b_1^3 + \frac{4}{3}b_1^2b_{s^2}\right)\left[I^R_{D\mathsf{P}_2}(k) + 2I^{\mathrm{bare},R}_{D\mathsf{P}_2}(k)\right]\end{aligned}$$

$$I^R_{DE}(k) \equiv W_R(k)\int\frac{\mathrm{d}^3q}{(2\pi)^3}W_R(q)W_R(|\mathbf{k}-\mathbf{q}|)P^{\mathrm{lin}}_{\mathrm{mm}}(q)P^{\mathrm{lin}}_{\mathrm{mm}}(|\mathbf{k}-\mathbf{q}|)D(\mathbf{q},\mathbf{k}-\mathbf{q})E(\mathbf{q},\mathbf{k}-\mathbf{q})$$

$$I^{\mathrm{bare},R}_{DE}(k) \equiv W_R(k)P^{\mathrm{lin}}_{\mathrm{mm}}(k)\int\frac{\mathrm{d}^3q}{(2\pi)^3}W_R(q)W_R(|\mathbf{k}-\mathbf{q}|)P^{\mathrm{lin}}_{\mathrm{mm}}(q)D(\mathbf{q},\mathbf{k}-\mathbf{q})E(\mathbf{q},-\mathbf{k})$$

- Covariance between cross-spectra at the same k

$$\mathrm{cov}\left(\hat{P}_{D[\delta_a^R],\delta_b^R}(k), \hat{P}_{E[\delta_a^R],\delta_b^R}(k)\right) = \frac{2}{N_{\mathrm{modes}}(k)}P^R_{bb}(k)I^{P^R_{aa}P^R_{aa}}_{DE}(k)$$

SIMULATIONS & RESULTS

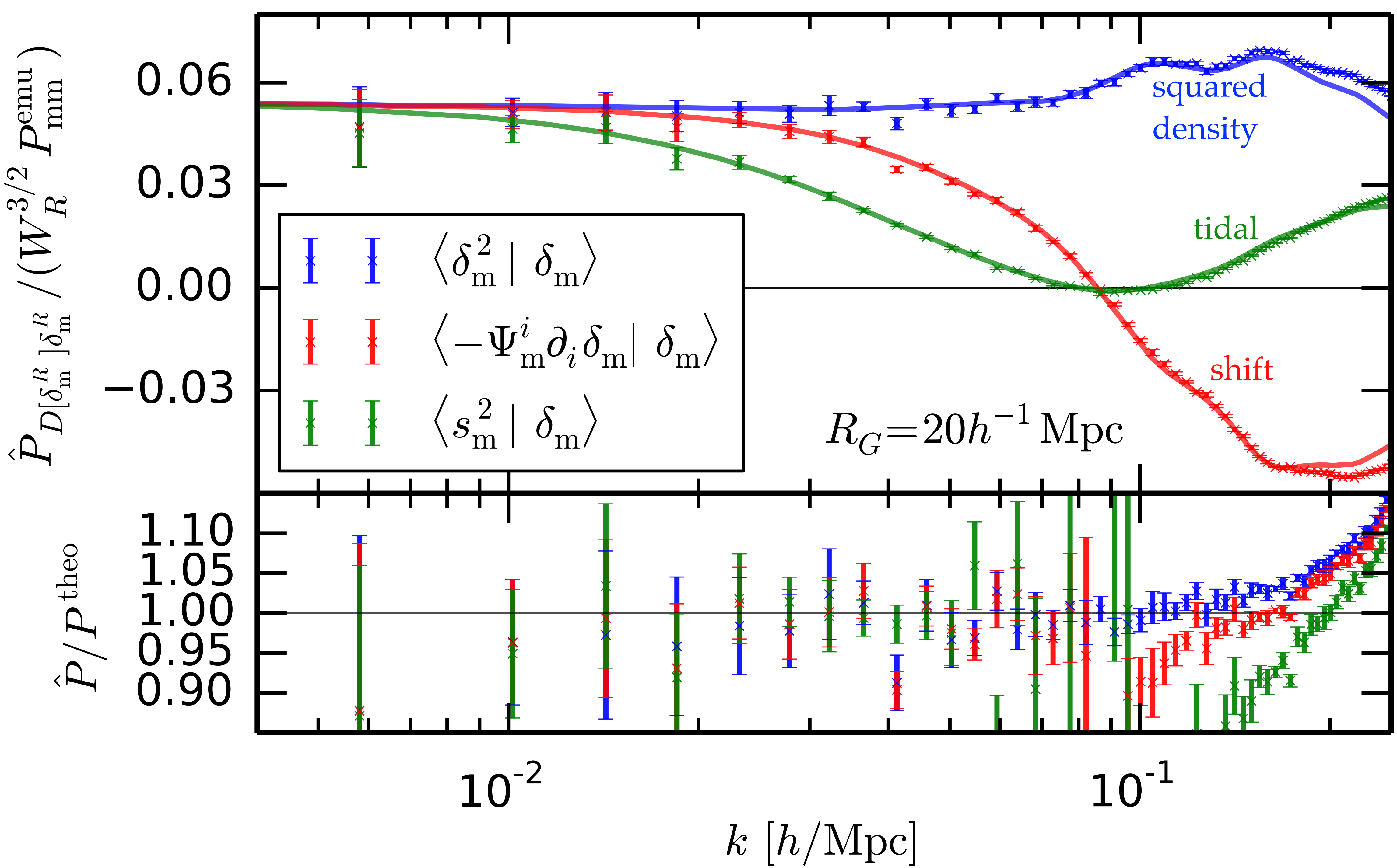
SIMULATIONS

Reid *et al.* 1404.3742
White *et al.* 1408.5435
White astro-ph/0207185

- **RunPB** sims from Martin White/Beth Reid
  - $L = 1380$ Mpc/h
  - 2048^3 DM particles or FoF (b=0.168) halos
  - 10 realizations
  - $z = 0.55$

DM CROSS-SPECTRA

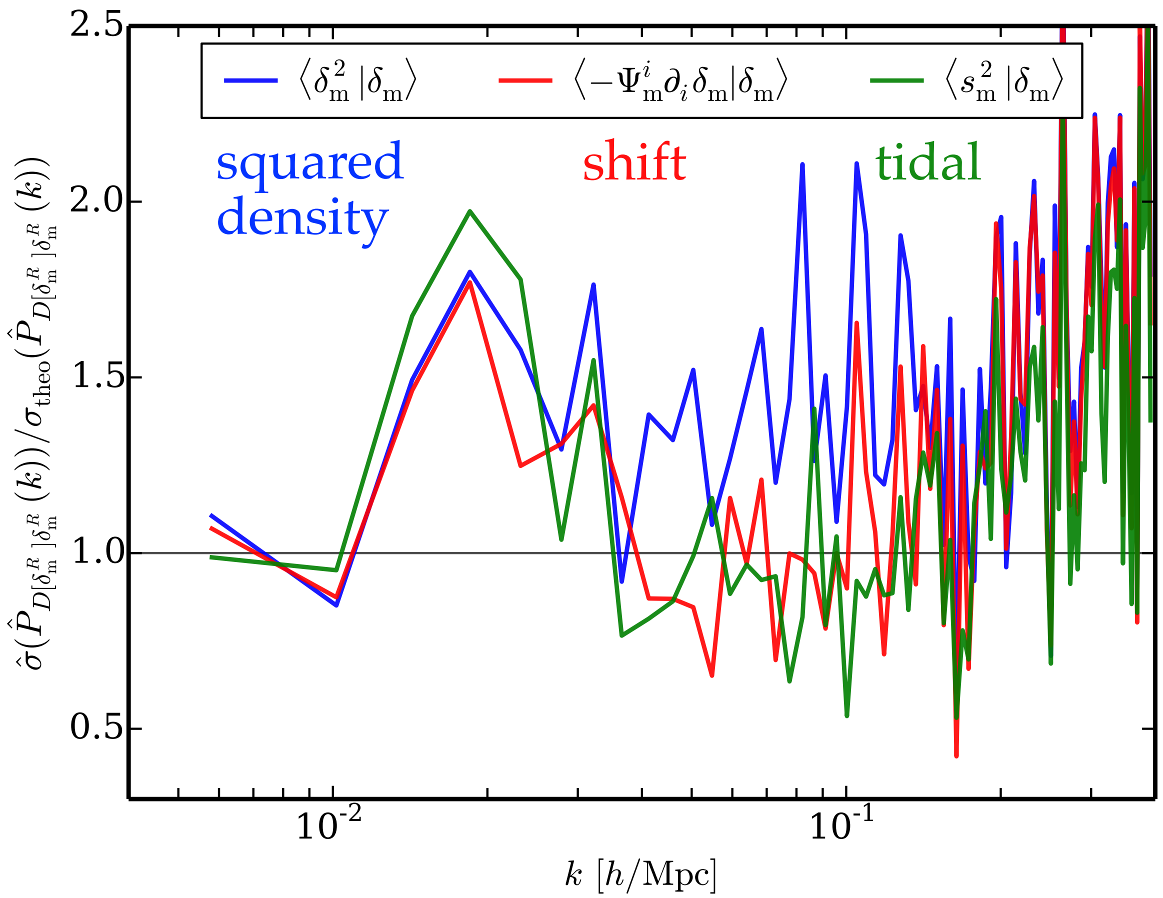
MS, Baldauf, Seljak, 1411.6595



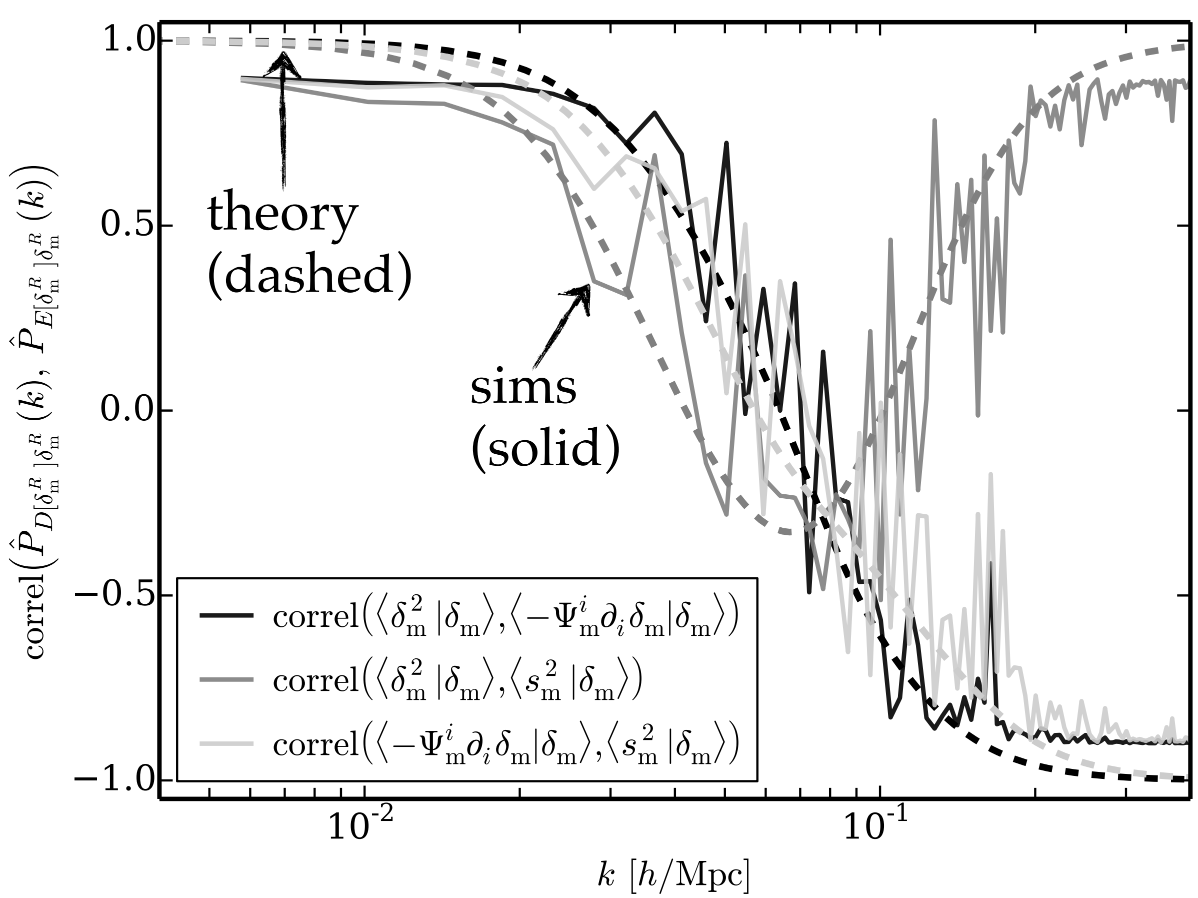
DM CROSS-SPECTRA: COVARIANCE

MS, Baldauf, Seljak, 1411.6595

Standard deviations of individual cross-spectra (sims divided by theory)



Correlations between different cross-spectra at the same k

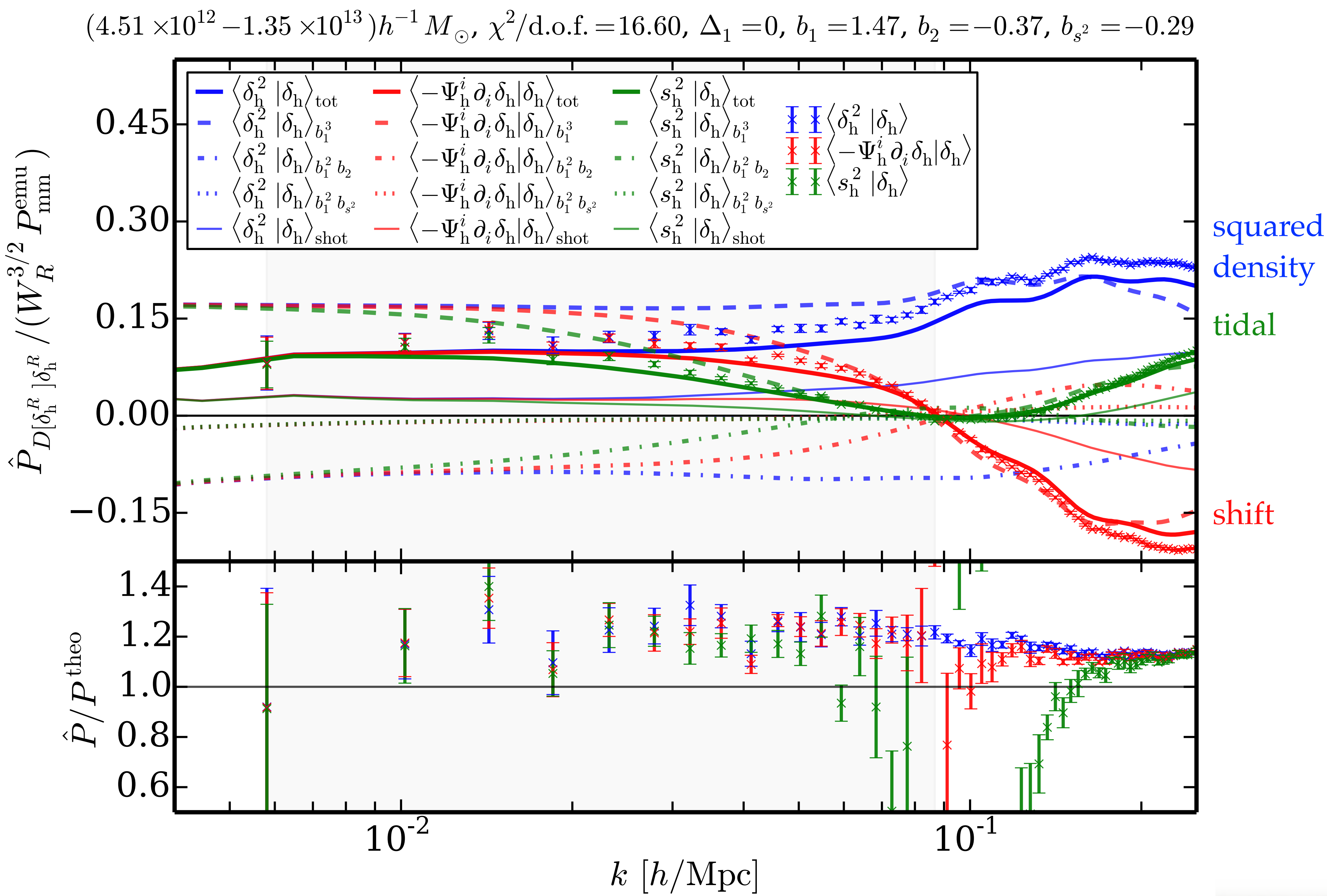


Perfectly (anti-)correlated at low k and high k, but uncorrelated at intermediate k

HALO CROSS-SPECTRA

MS, Baldauf, Seljak, 1411.6595

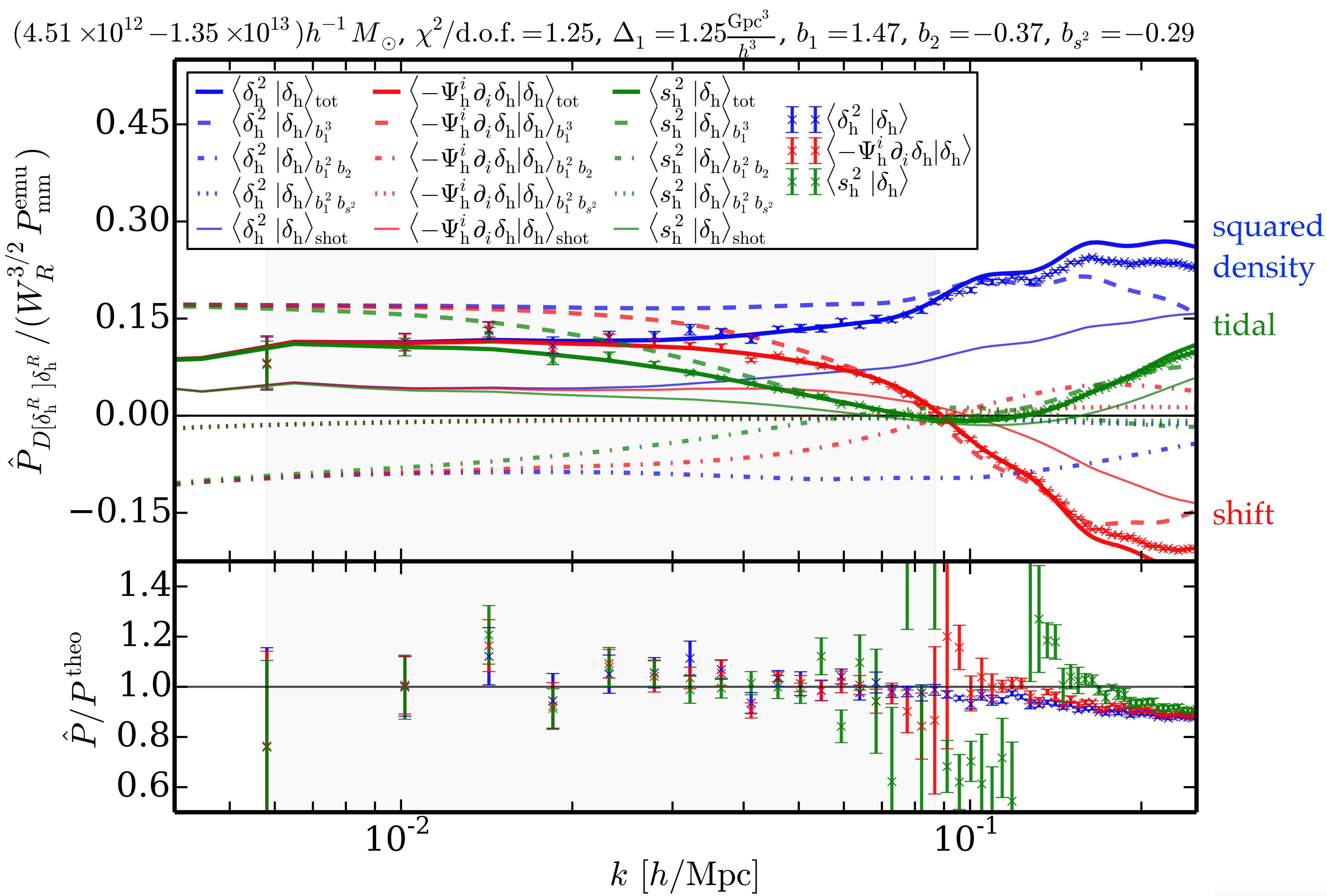
Assuming Poisson shot noise



HALO CROSS-SPECTRA

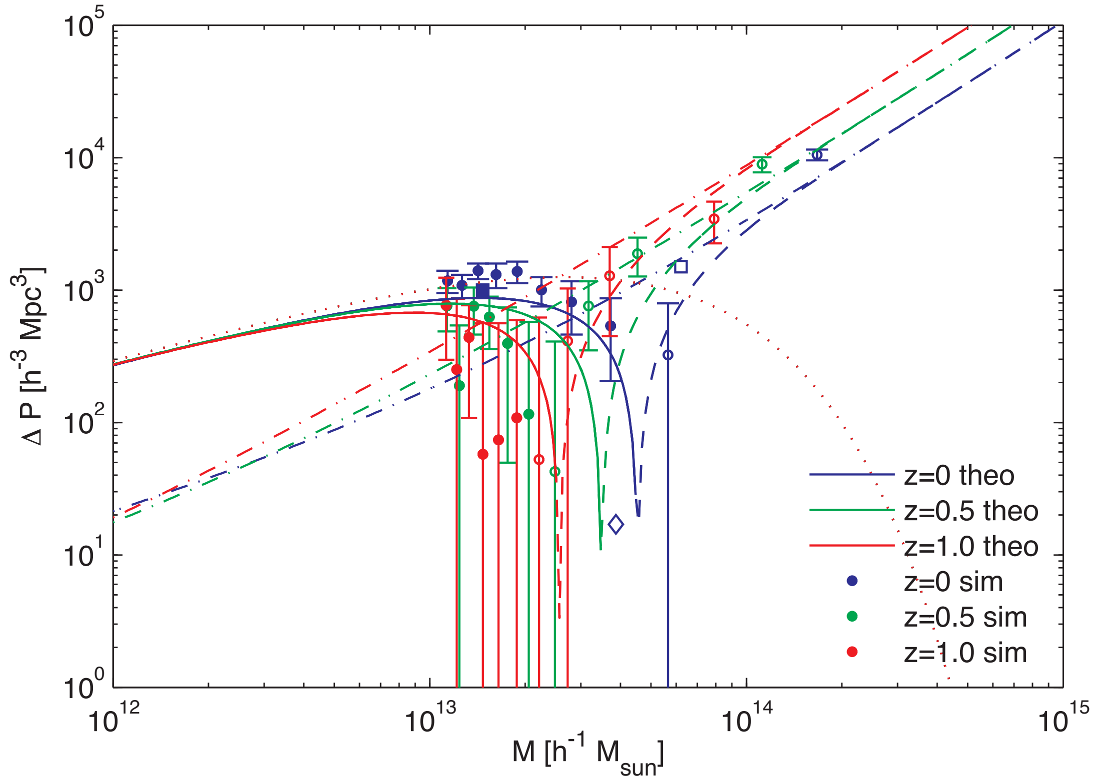
MS, Baldauf, Seljak, 1411.6595

Corrected shot noise (rescaling Poisson prediction by free A_{shot})

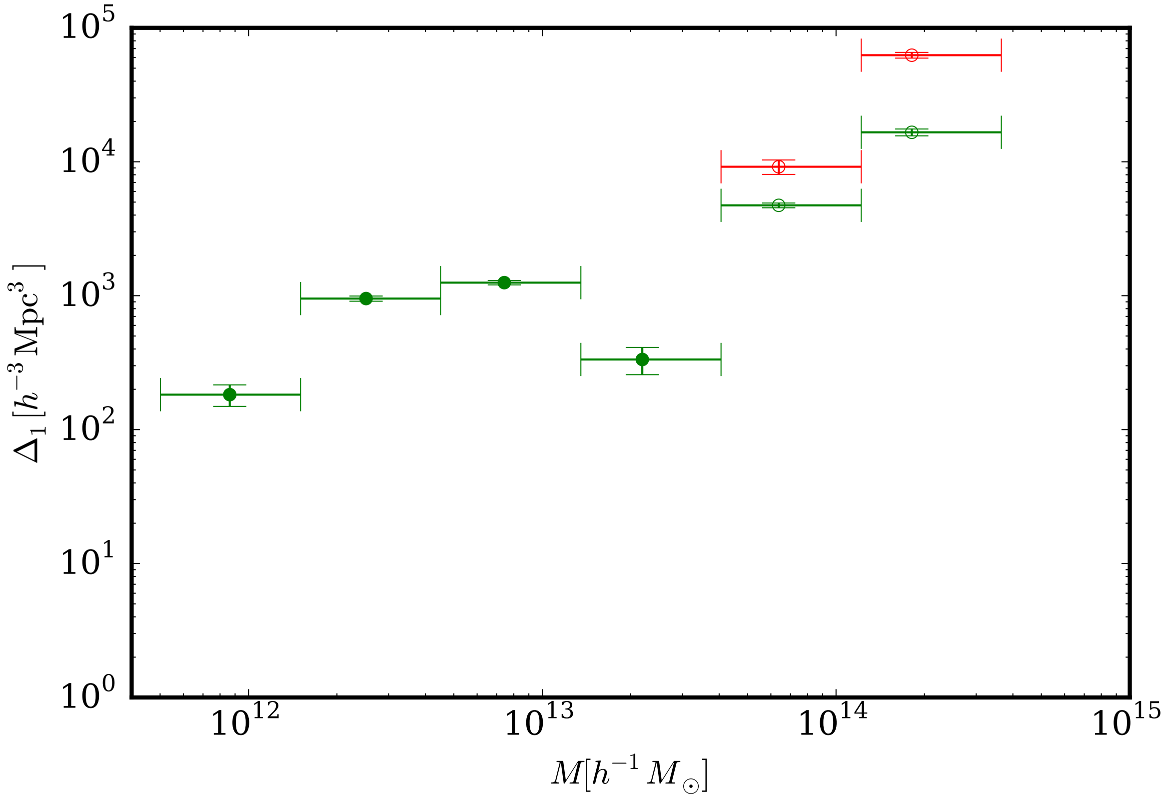


MASS DEPENDENCE OF SHOT NOISE CORRECTION

Correction to $1/n$ shot noise in ***P*(*k*)** (Baldauf *et al.* 1305.2917)



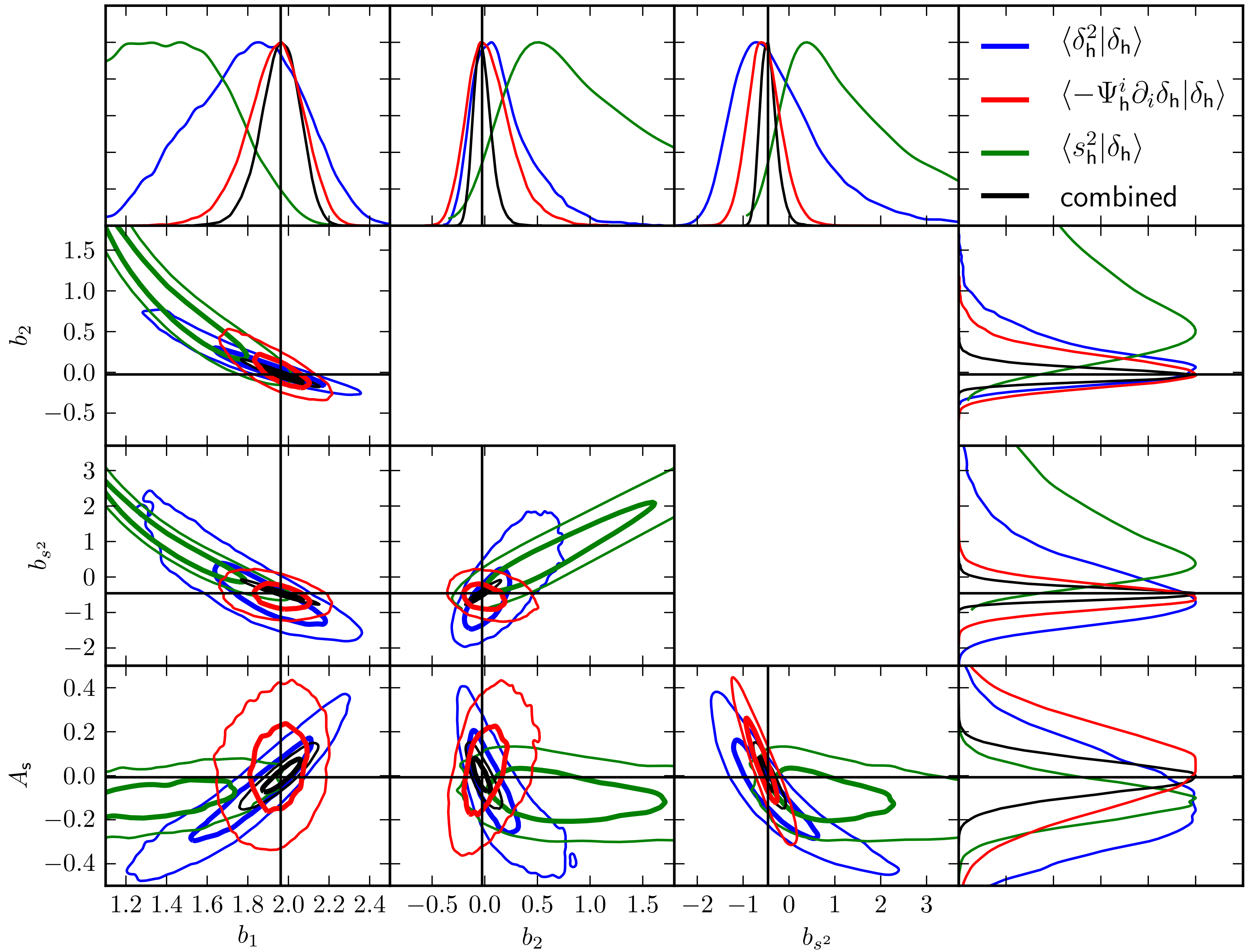
Correction to $P(k)/n$ part of shot noise in **bispectrum**



BIAS CONSTRAINTS

MS, Baldauf, Seljak, 1411.6595

Fit b_1, b_2, b_{s2}, $A_{\rm shot}$ from *hhh* cross-spectra (only 3-point, no 2-point)



squared density
shift
tidal

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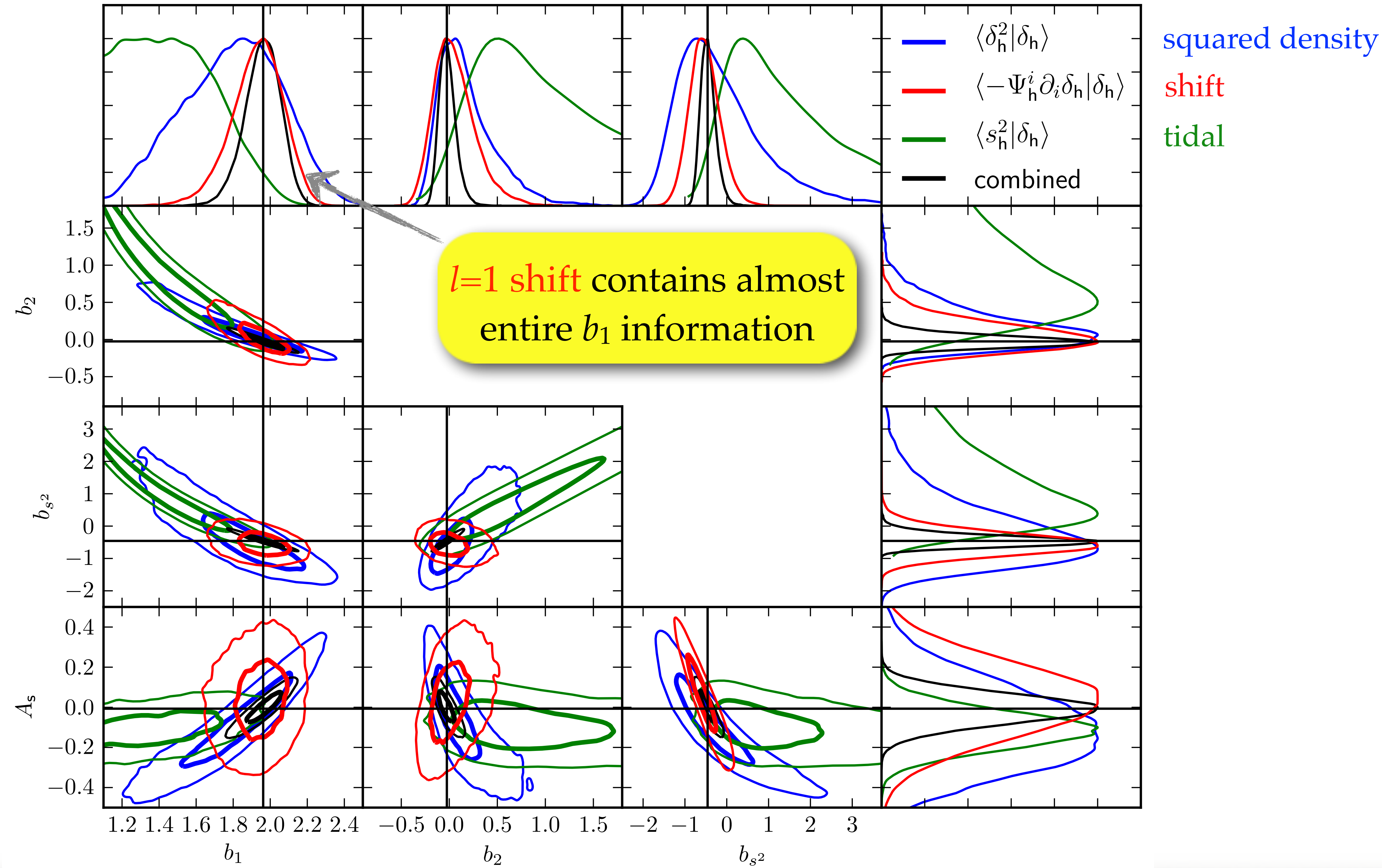

CONCLUDING REMARKS (PART I)

COMPARISON WITH TRADITIONAL BISPECTRUM ESTIMATION

MS, Baldauf, Seljak, 1411.6595

- **Advantages** of cross-spectra over estimating bispectrum for individual triangles
  - *Simple*
  - *Nearly optimal* (contains entire bispectrum information)
  - *Fast* (like power spectrum estimation)
  - Depends on just single *k*, i.e. covariances are easy

Position-dependent power spectrum (Chiang++) is only sensitive to squeezed limit where grav. signal vanishes

- **Challenges**
  - Redshift space distortions: messier but doable (ideas/tricks welcome!)
  - Not optimal beyond tree level and beyond weakly non-Gaussian fields
    (could be improved with better theory or by combining with modal estimators that are sensitive to arbitrary bispectra)

WHAT DO CMB PEOPLE DO?

- **CMB bispectrum estimation**
  - 2001: individual triangles of COBE data ($l_{\max}$=20) — Komatsu *et al.* 2001
  - since 2003: **KSW** f_{NL} estimators for separable bispectrum templates (now standard) — Komatsu+Spergel +Wandelt 2003
  - **CMB lensing** trispectrum:
    - ➟ never used individual quadrilaterals, but always max-likeli estimators for known trispectrum shape
    - ➟ reconstructed lensing potential is $\widehat{\nabla\phi} \propto T\nabla T$
    - ➟ trispectrum = auto-power of this quadratic field
  - ISW-lensing bispectrum: cross-spectrum of $\hat{\phi} \sim T^2$ and T — Lewis *et al.* 2011, Planck 2013 XIX (ISW)
  - modal estimators (sensitive to arbitrary bispectra — reduce to our cross-spectra if halo bispectrum contributions are used as basis shapes) — Fergusson+Shellard +Liguori 2009-2014, Planck 2013 XXIV (NG)

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LSS methods at the level where CMB was 2001

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Komatsu+Spergel +Wandelt 2003

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LSS methods at the level where CMB was 2001

Cross-spectrum method is *first step* to modernize this for LSS

but it's really only a first step:

our method is not yet ready for real data,

while traditional method has been applied to real data

(most recently by Gil-Marin *et al.* 2014)

CONCLUSIONS (PART I)

MS, Baldauf, Seljak, 1411.6595

- Explored new estimators to measure bispectrum parameters in a nearly optimal way using cross-spectra of 3 quadratic fields with the density

$$\hat{P}_{\delta^2,\delta}(k) \sim \sum_{\mathbf{k},|\mathbf{k}|=k} [\delta^2](\mathbf{k})\delta(-\mathbf{k}),$$

$$\hat{P}_{-\Psi^i\partial_i\delta,\delta}(k) \sim \sum_{\mathbf{k},|\mathbf{k}|=k} [-\Psi^i\partial_i\delta](\mathbf{k})\delta(-\mathbf{k})$$

$$\hat{P}_{s^2,\delta}(k) \sim \sum_{\mathbf{k},|\mathbf{k}|=k} [s^2](\mathbf{k})\delta(-\mathbf{k}).$$

- Simulations agree with theory on large scales
- Linear halo bias b_1 mostly determined by shift term cross density
- Shot noise requires non-Poissonian corrections
- *Future*: e.g. RSDs, model to higher k, apply to real data

PART II

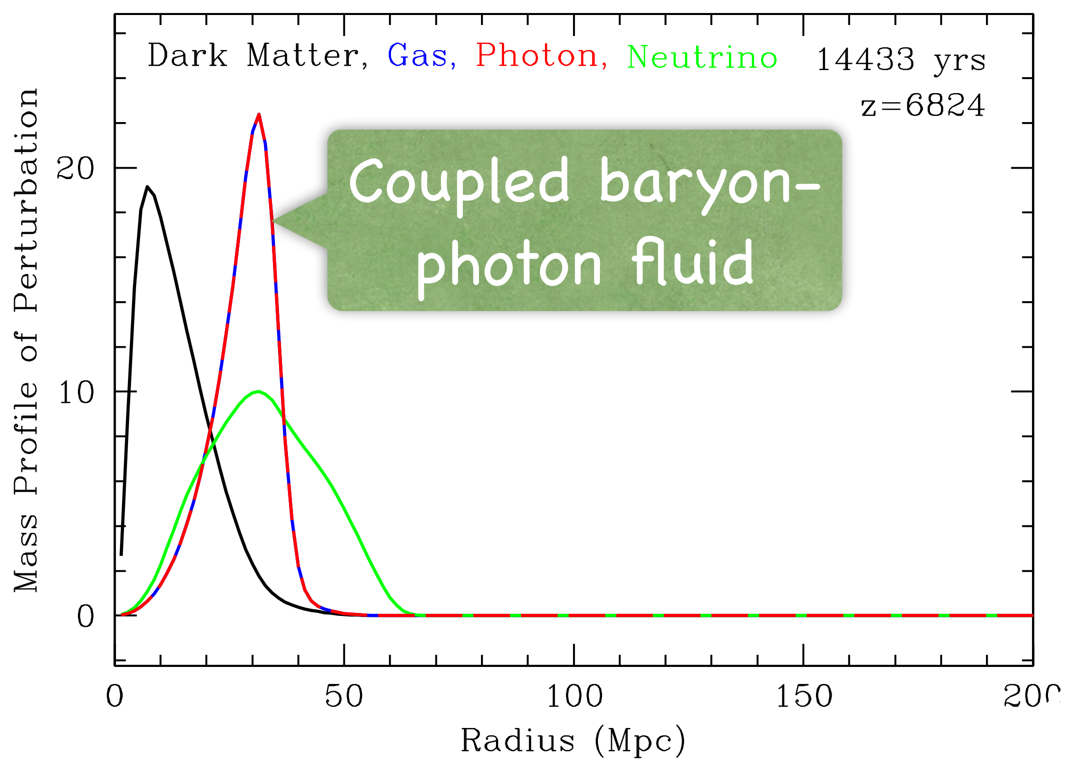
EULERIAN BAO RECONSTRUCTIONS
&
N-POINT STATISTICS

ARXIV:1508.06972

W/ Y. FENG, F. BEUTLER, B. SHERWIN, M.Y. CHU

BARYONIC ACOUSTIC OSCILLATIONS (BAO)

Eisenstein / Seo / White 2006



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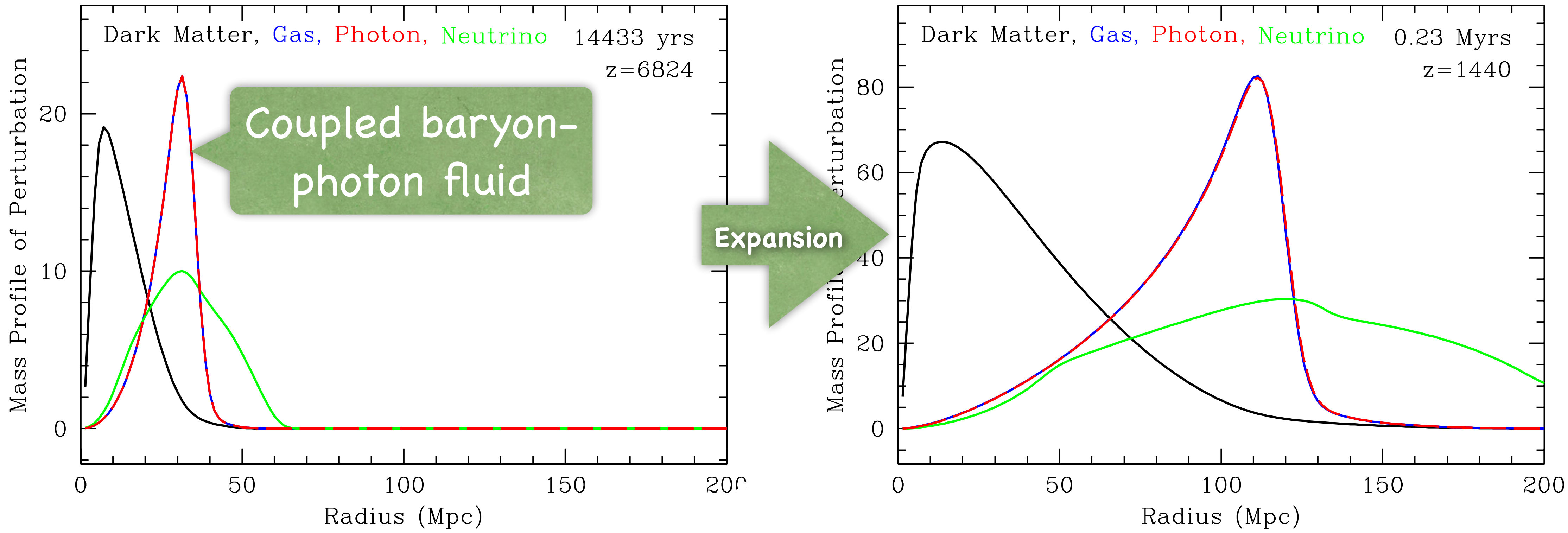
Eisenstein / Seo / White 2006



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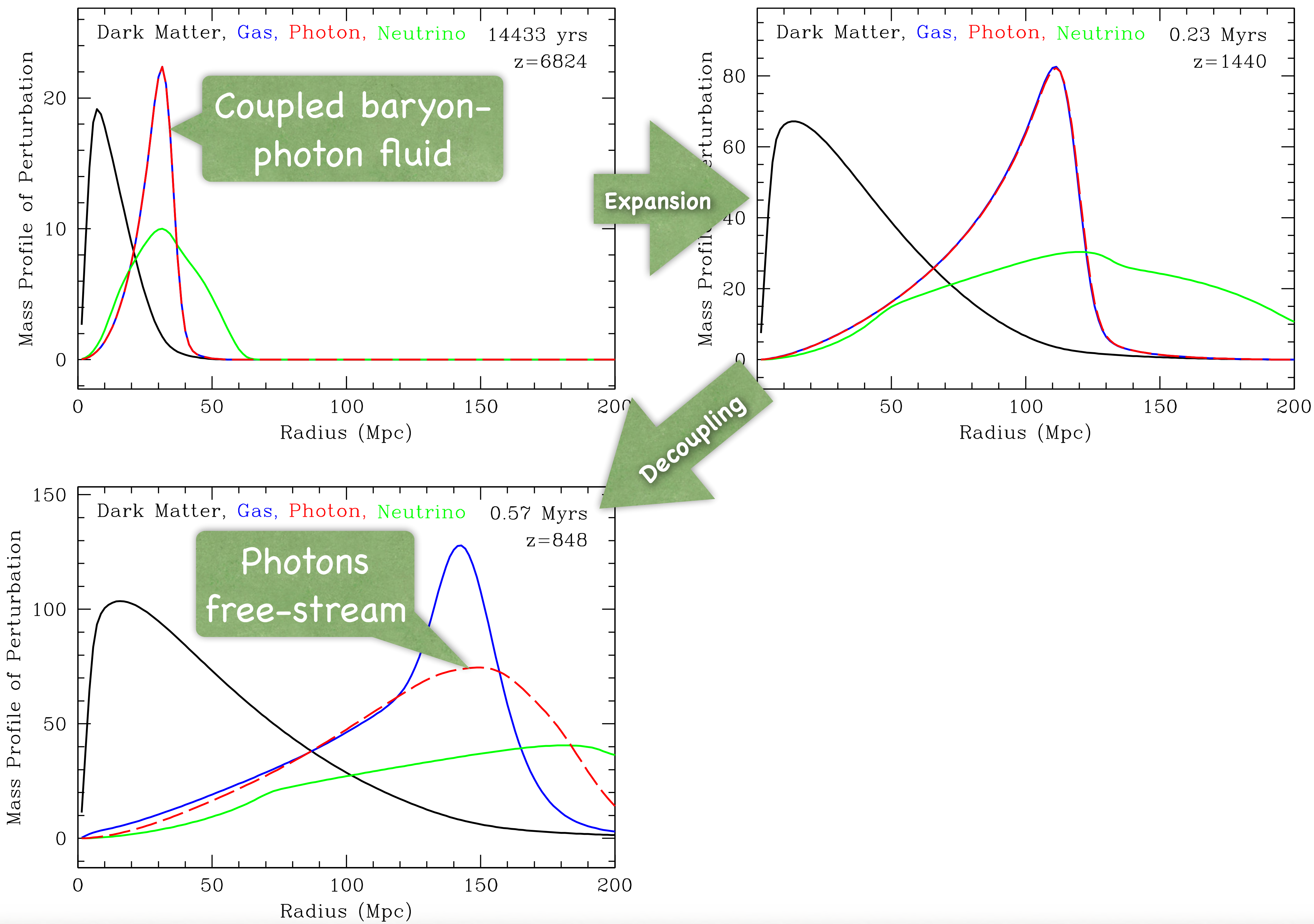

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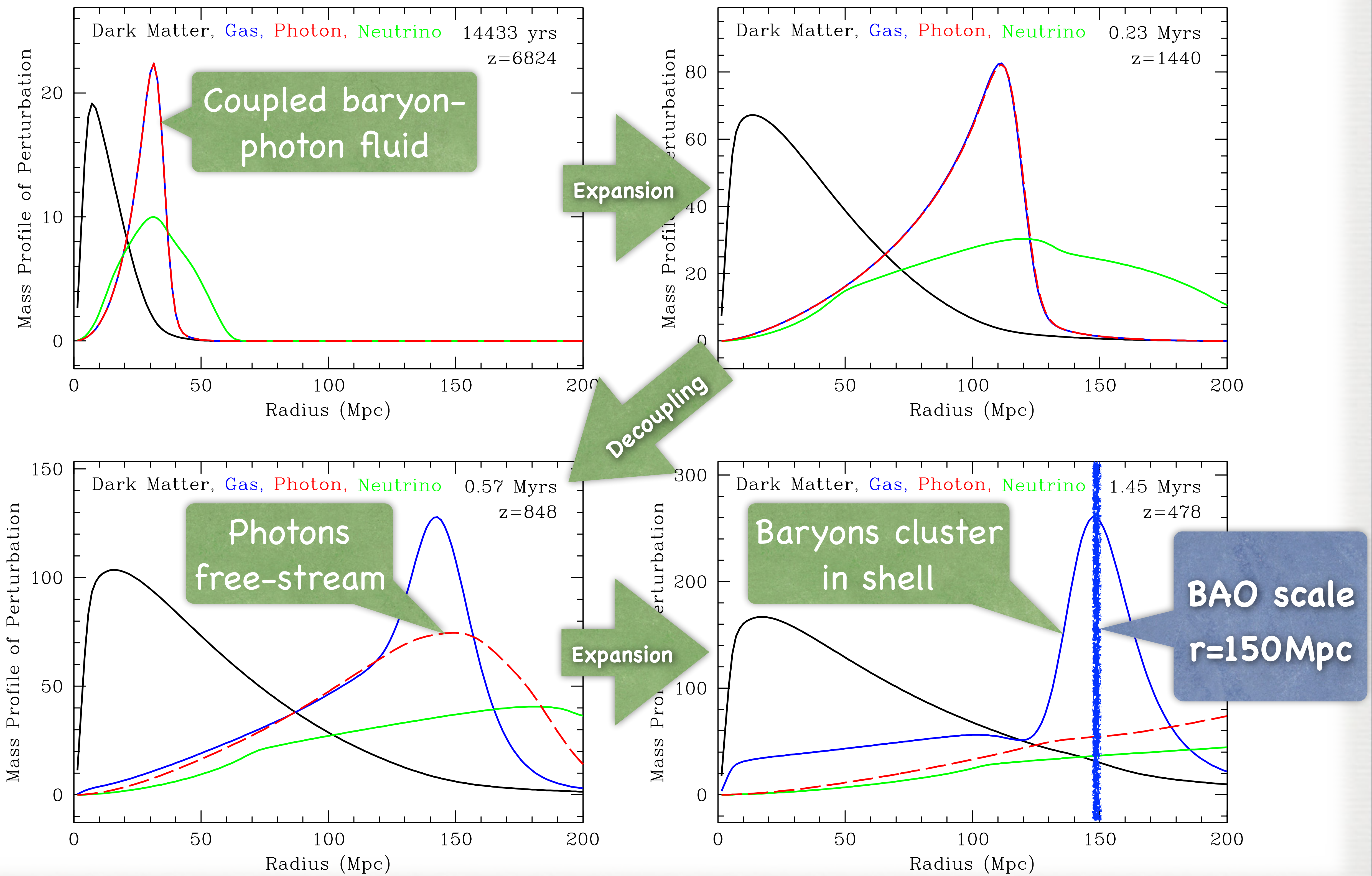
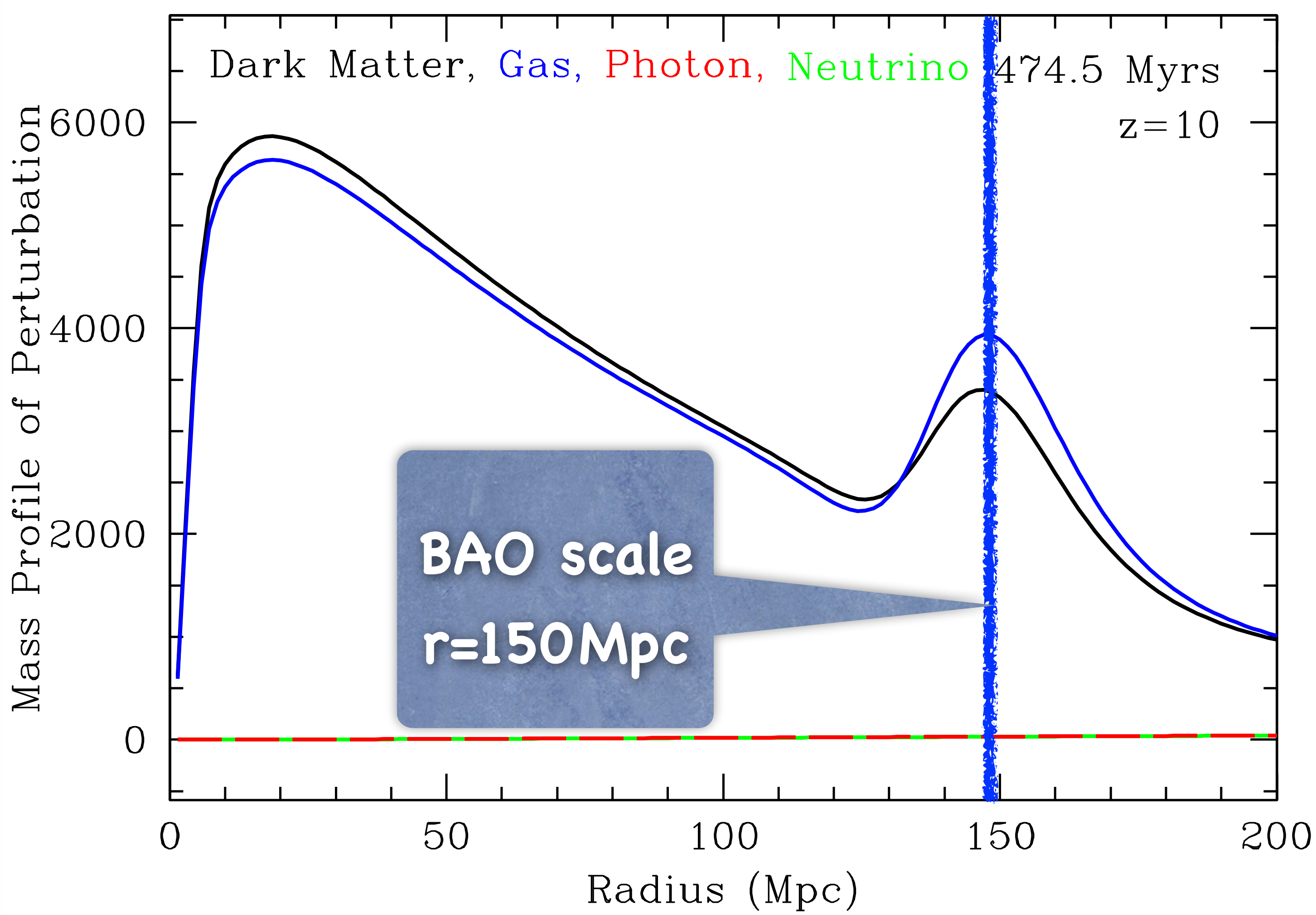
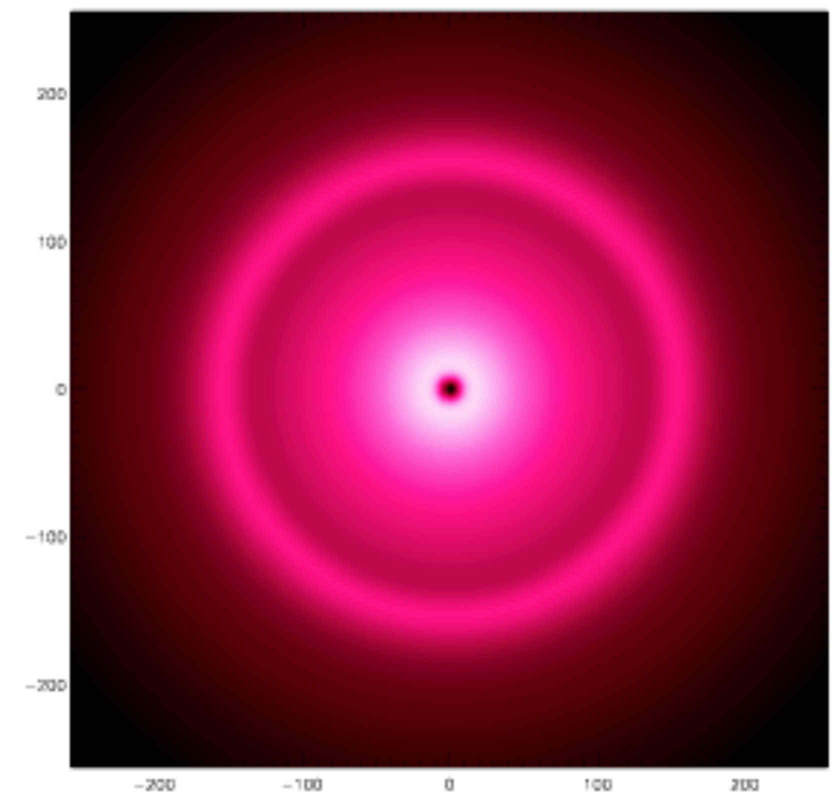


BARYONIC ACOUSTIC OSCILLATIONS (BAO)

Eisenstein / Seo / White 2006

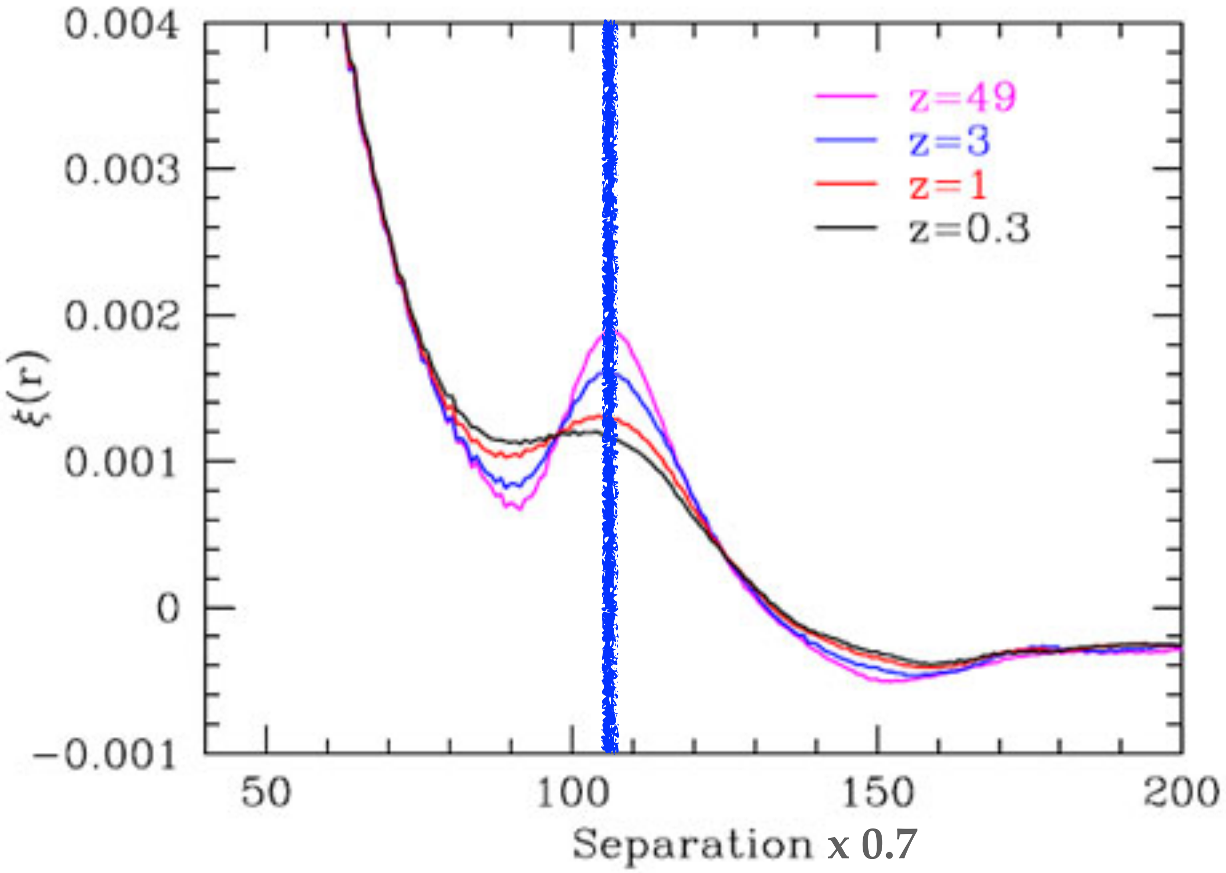
Expansion + clustering



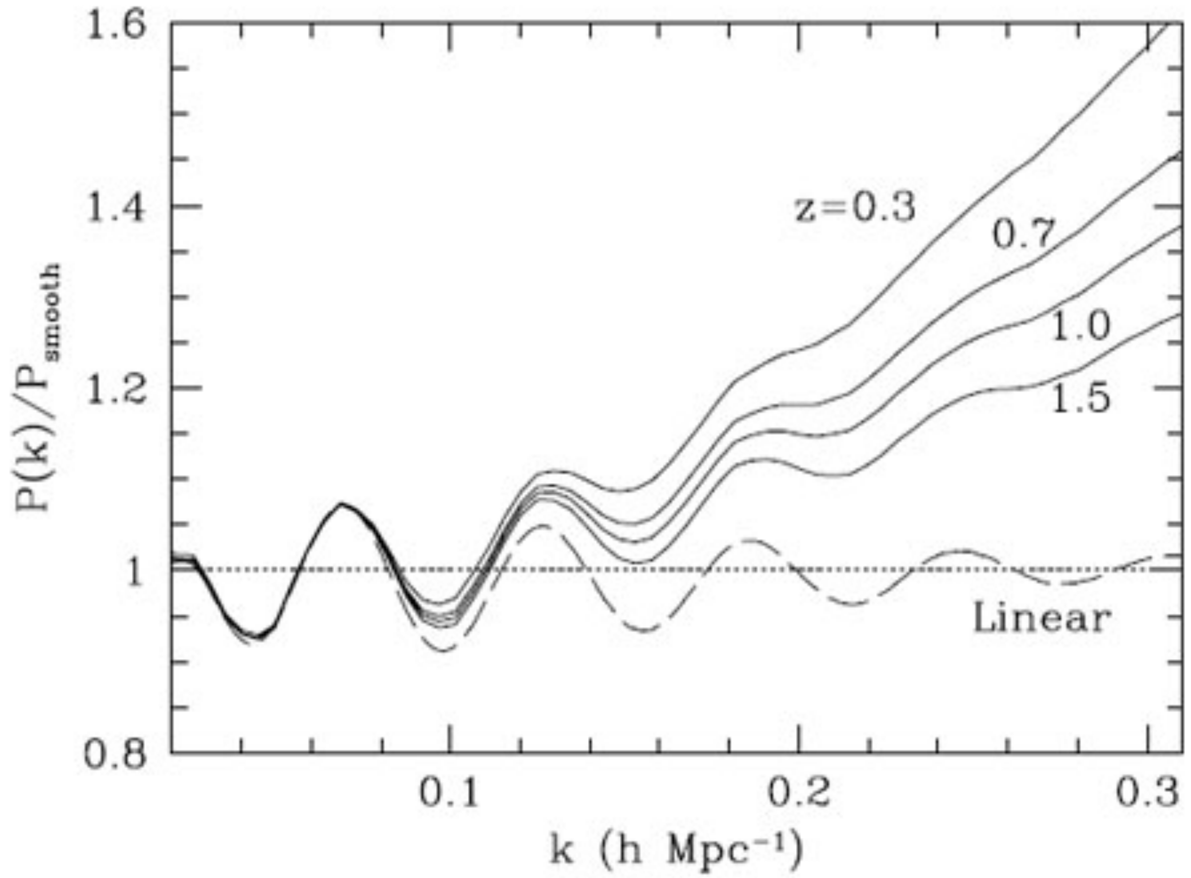


BAO IMPRINT ON LSS

Correlation function $\xi(r)$: BAO bump



Power spectrum *P(k):* BAO wiggles



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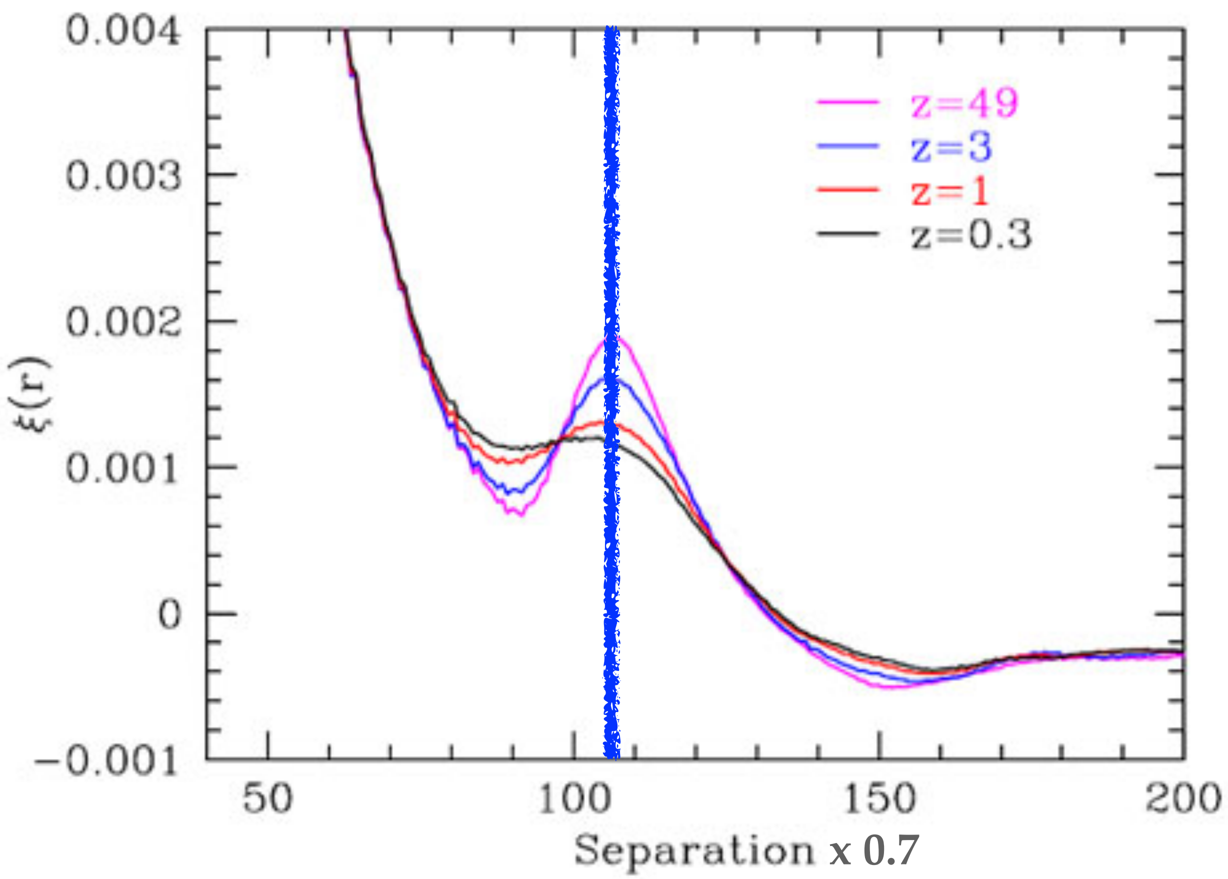

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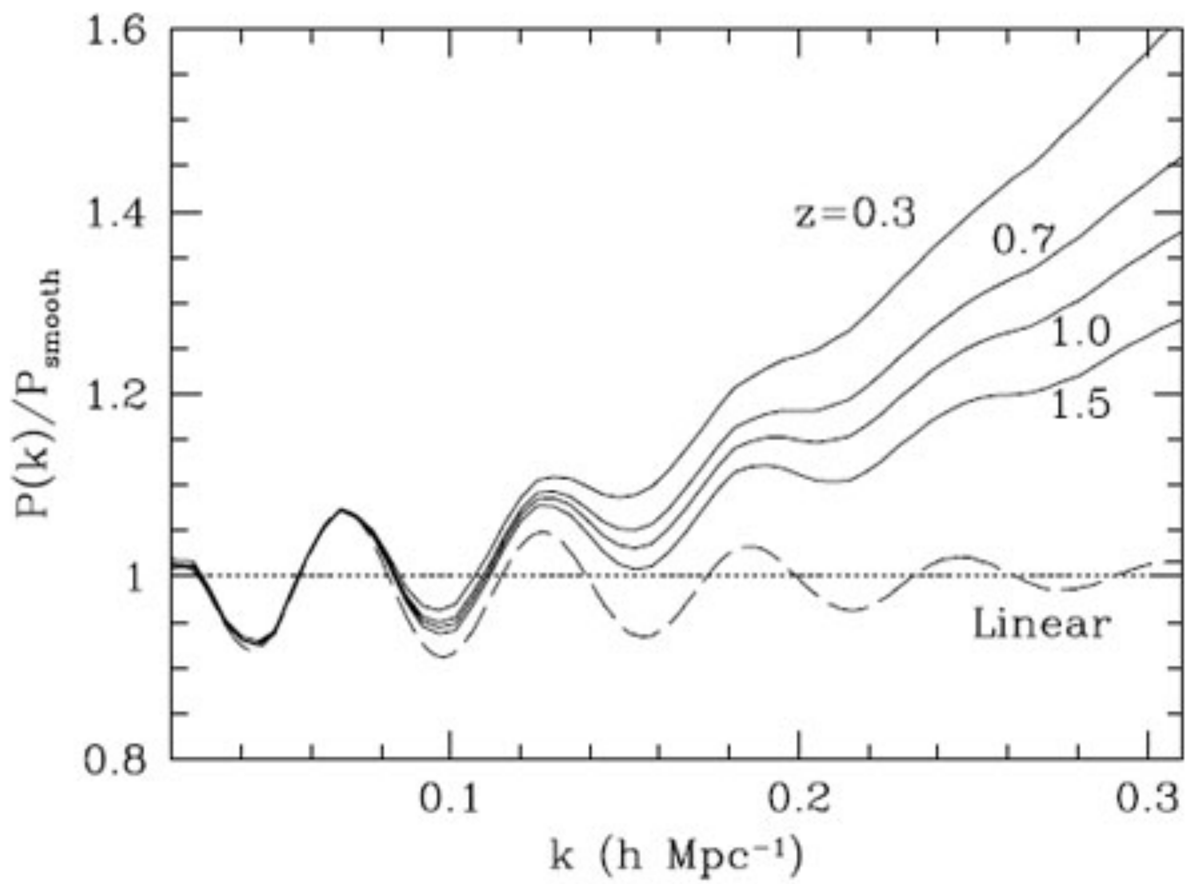


- ***Non-linearities:***
  - Add small-scale power
  - Smear out BAO peak, i.e. degrade BAO information
  - Slightly shift BAO scale

BAO RECONSTRUCTION

Eisenstein et al. 2006
Padmanabhan et al. 2008

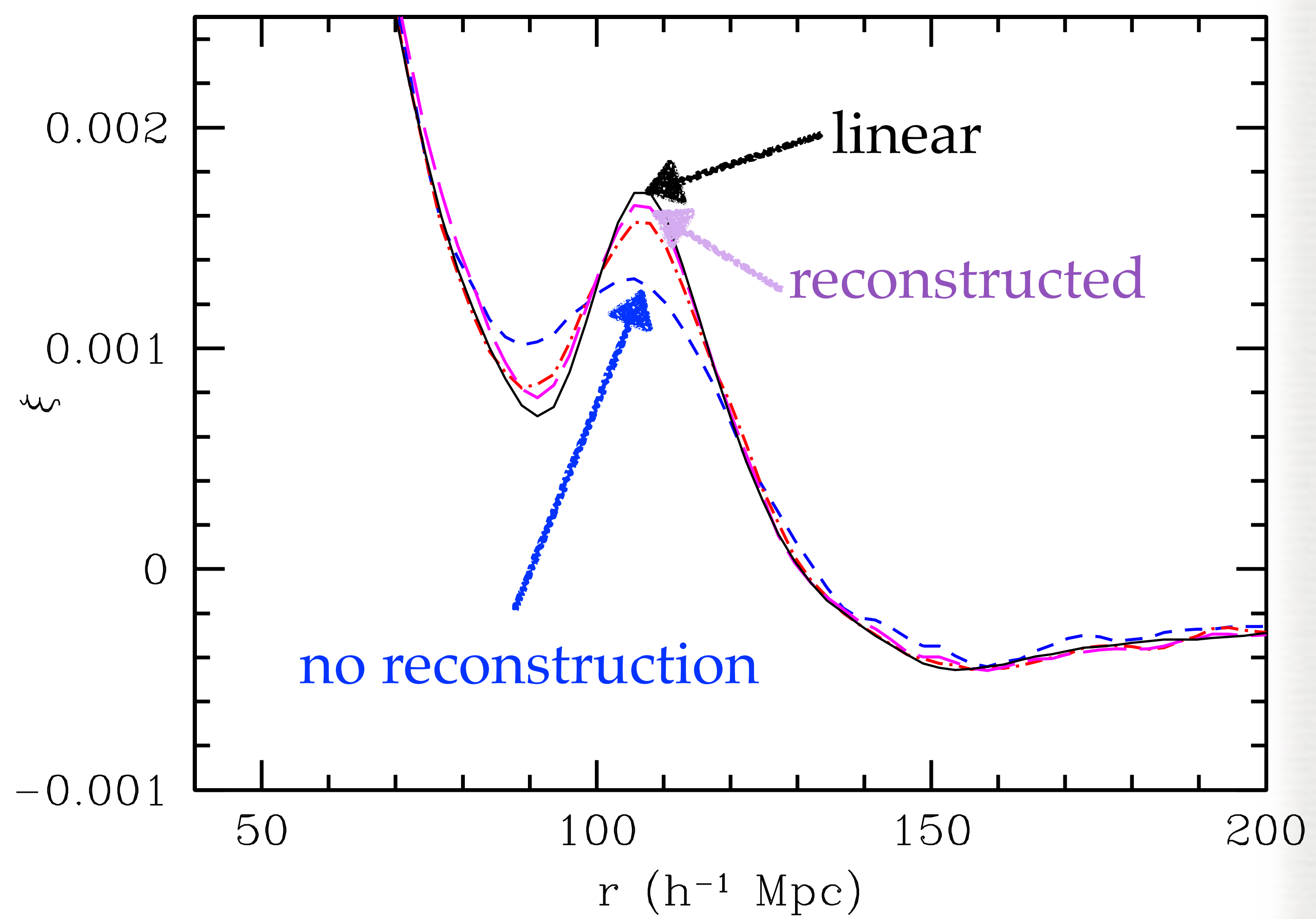
- *Goal:* Restore linear modes to increase BAO information

- *Method:* Undo large-scale flows that broaden the BAO peak:
    - Calculate large-scale displacement/velocity field with Zeldovich approximation

$$\mathbf{s}(\mathbf{k}) = -\frac{i\mathbf{k}}{k^2} W_R(k)\delta(\mathbf{k})$$

    - Displace clustered and random catalog by this
        - ➡ get `displaced' and `shifted' densities δ_d and δ_s
    - Reconstructed density = δ_d - δ_s

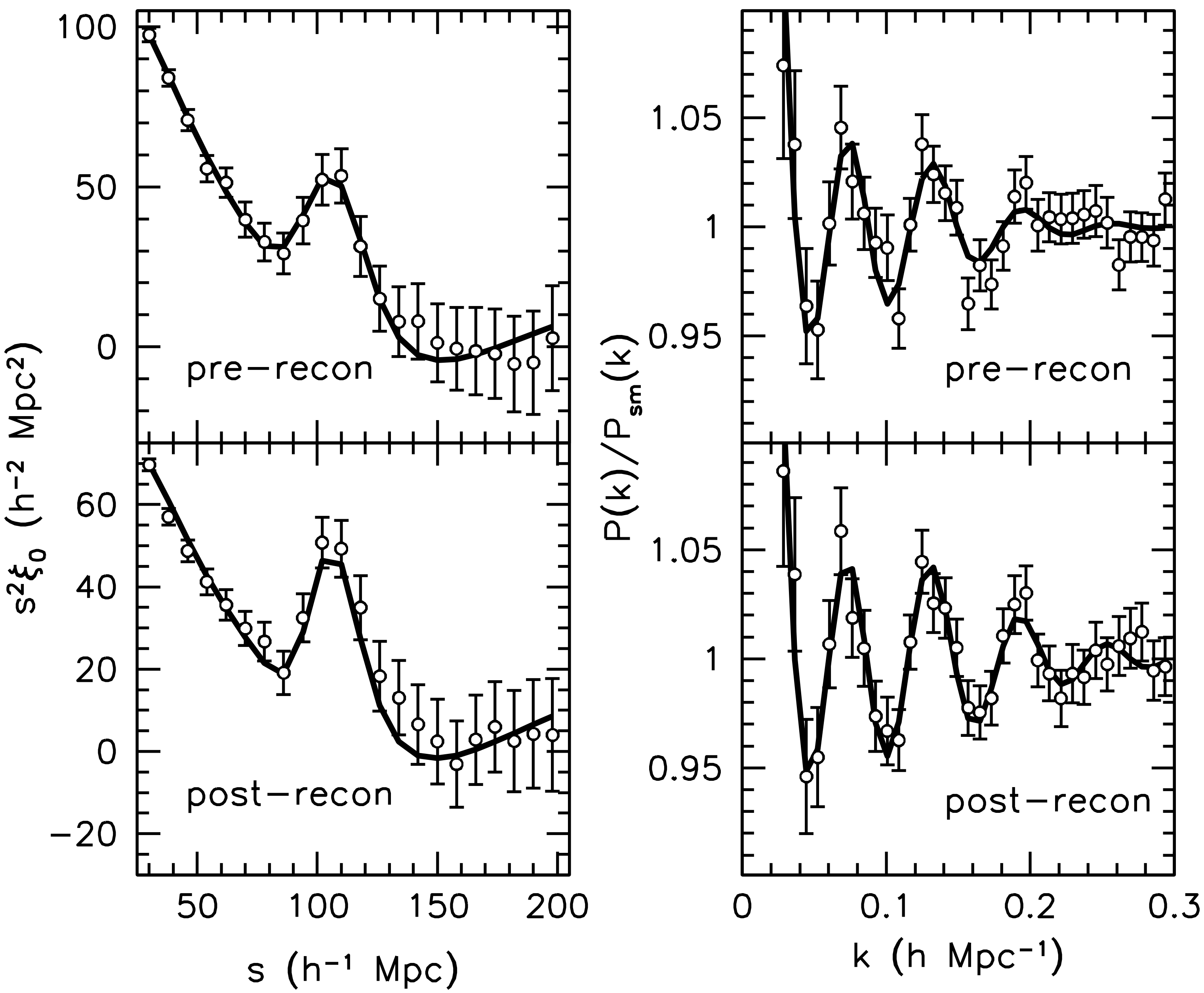
Simulations



BAO RECONSTRUCTION

Padmanabhan et al. 2012
Anderson et al. 2013

- Works great in practice:
  For BOSS DR11, BAO signal-to-noise improved by ~50%,
  achieving sub-percent BAO scale constraint



Anderson et al. 2013
(BOSS DR11)

NEW RECONSTRUCTION ALGORITHMS

NEW ALGORITHMS: WHY?

- Motivations to dig deeper & explore new reconstruction algorithms:
    - Where does the new BAO information come from?
    - Can we derive reconstruction algorithm(s) better?
    - Can we do better in terms of BAO information?
    - Can we reduce intermixing of model and data when reconstructing?

      (e.g. standard reconstruction makes assumptions on *f*, galaxy bias, gravity when transforming data)

- Tried to address some of these points in recent paper (1508.06972)

NEW ALGORITHMS: CATEGORIES

- Useful to consider two categories of reconstruction algorithms:

$\boxed{\text{Eulerian reconstructions}}$

- Never displace any objects
- Work only with Eulerian density
- E.g. $\delta_{\rm rec} = \delta - \delta^2$

NEW ALGORITHMS: CATEGORIES

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Eulerian reconstructions

- Never displace any objects
- Work only with Eulerian density
- E.g. $\delta_{\rm rec} = \delta - \delta^2$

Lagrangian reconstructions

- Displace objects at some stage of the algorithm
- E.g. standard algorithm (Eisenstein et al. 2006)

NEW EULERIAN ALGORITHM I

MS et al. 1508.06972

- Eulerian growth-shift (EGS) reconstruction
  - Go back in time

$$\delta_{\mathrm{rec}}(\mathbf{x}) = \delta(\mathbf{x}, \eta - \Delta\eta) \approx \delta(\mathbf{x}, \eta) - \Delta\eta\, \partial_\eta \delta(\mathbf{x}, \eta)$$

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  - Get density time derivative nonperturbatively from nonlinear continuity equation

$$\partial_\eta \delta + \nabla \cdot [(1+\delta)\mathbf{v}] = 0$$

$$\Rightarrow \quad \partial_\eta \delta = -\nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla\delta - \delta \nabla \cdot \mathbf{v}$$

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$$\Rightarrow \partial_\eta \delta = -\nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla \delta - \delta \nabla \cdot \mathbf{v}$$

  - Approximate velocity in terms of smoothed density δ_R (using linear relation)

$$\mathbf{v}(\mathbf{k}) \approx -\mathbf{s}(\mathbf{k}) \equiv \frac{i\mathbf{k}}{k^2} \delta_R(\mathbf{k}) \quad \Rightarrow \nabla \cdot \mathbf{v} = \delta_R$$

$$\Rightarrow \delta^{\mathrm{rec}}_{\mathrm{EGS}}(\mathbf{x}) = \delta(\mathbf{x}) - \underbrace{\mathbf{s}(\mathbf{x}) \cdot \nabla \delta(\mathbf{x})}_{\text{shift}} - \underbrace{\delta(\mathbf{x}) \delta_R(\mathbf{x})}_{\text{growth}}$$

- Not reverting linear time evolution
- Not displacing any objects, i.e. Eulerian
- Nonperturbative in δ, linear in velocity

NEW EULERIAN ALGORITHM I

MS et al. 1508.06972

- Eulerian growth-shift (EGS) reconstruction
  - **Reconstructed power spectrum:**

$$\delta^{\mathrm{rec}}_{\mathrm{EGS}} = \delta - \delta^2 - \mathbf{s}\cdot\nabla\delta$$

$$\Rightarrow \langle \delta^{\mathrm{rec}}_{\mathrm{EGS}} | \delta^{\mathrm{rec}}_{\mathrm{EGS}} \rangle = \langle \delta | \delta \rangle \qquad \text{2-point}$$

$$-2\left\langle \delta^2 | \delta \right\rangle - 2\left\langle \mathbf{s}\cdot\nabla\delta | \delta \right\rangle \qquad \text{3-point}$$

$$+\left\langle \delta^2 | \delta^2 \right\rangle + \left\langle \mathbf{s}\cdot\nabla\delta | \mathbf{s}\cdot\nabla\delta \right\rangle + 2\left\langle \delta^2 | \mathbf{s}\cdot\nabla\delta \right\rangle \qquad \text{4-point}$$

- ➡ Automatically combines 2-, 3- and 4-point of the unreconstructed density
- ➡ Explicitly see how reconstruction exploits 3- and 4-point BAO information
- ➡ Cross-spectra are the same as in part I of the talk, i.e. nearly-optimal
- ➡ Get similar combinations for all our other Eulerian reconstructions

NEW EULERIAN ALGORITHM II

MS et al. 1508.06972

- Eulerian F2 (EF2) reconstruction

  - Second-order density

$$\delta^{(2)}(\mathbf{x}) = \underbrace{\frac{17}{21}\delta_0^2(\mathbf{x})}_{\text{growth}} - \underbrace{\mathbf{\Psi}_0(\mathbf{x})\cdot\nabla\delta_0(\mathbf{x})}_{\text{shift}} + \underbrace{\frac{4}{21}K_0^2(\mathbf{x})}_{\text{tidal}}$$

  - Remove this from the full density

$$\delta^{\mathrm{rec}}_{\mathrm{EF2}}(\mathbf{x}) \equiv \delta(\mathbf{x}) - \underbrace{\frac{17}{21}\delta_R(\mathbf{x})\delta(\mathbf{x})}_{\text{growth}} - \underbrace{\mathbf{s}(\mathbf{x})\cdot\nabla\delta(\mathbf{x})}_{\text{shift}} - \underbrace{\frac{4}{21}K_R^2(\mathbf{x})}_{\text{tidal}}$$

  - More formal derivation using Newton-Raphson iteration:
    Find linear density compatible with a given observed nonlinear density, i.e. solve
    $f[\delta_0] \equiv \delta_0 + F_2[\delta_0] - \delta_{\mathrm{obs}} = 0$ for δ_0

NEW LAGRANGIAN ALGORITHMS

MS et al. 1508.06972

- Lagrangian Random-random (LRR) reconstruction
  - 2LPT theory for clustered and random catalogs displaced by negative or positive Zeldovich displacement: $\delta_d[\mathbf{s}]$, $\delta_d[-\mathbf{s}]$, $\delta_s[\mathbf{s}]$, $\delta_s[-\mathbf{s}]$
  - Only 2 of 6 possible combinations suppress 2nd order nonlinearities:
    - Lagrangian growth-shift (LGS) reconstruction (= standard algorithm)

$$\delta^{\mathrm{rec}}_{\mathrm{LGS}} = \delta_d[\mathbf{s}] - \delta_s[\mathbf{s}]$$

    - Lagrangian random-random (LRR) reconstruction:

$$\boxed{\delta^{\mathrm{rec}}_{\mathrm{LRR}} = \delta - \frac{1}{2}\left\{\delta_s[\mathbf{s}] + \delta_s[-\mathbf{s}]\right\}}$$

NEW LAGRANGIAN ALGORITHMS

MS et al. 1508.06972

- Lagrangian F2 (LF2) reconstruction
  - Unique combination of LRR and standard LGS algorithms reverses full second-order density (up to smoothing)

$$\delta^{\rm rec}_{\rm LF2} = \frac{3}{7}\delta^{\rm rec}_{\rm LGS} + \frac{4}{7}\delta^{\rm rec}_{\rm LRR}$$

OVERVIEW

MS et al. 1508.06972

- All reconstruction algorithms

	Eulerian	Lagrangian
Growth-Shift	$\delta^{\rm rec}_{\rm EGS}(\mathbf{x}) = \delta(\mathbf{x}) - \delta_R(\mathbf{x})\delta(\mathbf{x}) - \mathbf{s}(\mathbf{x})\cdot\nabla\delta(\mathbf{x})$	$\delta^{\rm rec}_{\rm LGS} = \delta_d[\mathbf{s}] - \delta_s[\mathbf{s}]$
Full F2	$\delta^{\rm rec}_{\rm EF2}(\mathbf{x}) = \delta(\mathbf{x}) - \frac{17}{21}\delta_R(\mathbf{x})\delta(\mathbf{x}) - \mathbf{s}(\mathbf{x})\nabla\delta(\mathbf{x}) - \frac{4}{21}K^2_R(\mathbf{x})$	$\delta^{\rm rec}_{\rm LF2} = \frac{3}{7}\delta^{\rm rec}_{\rm LGS} + \frac{4}{7}\delta^{\rm rec}_{\rm LRR}$
Random-Random	$\delta^{\rm rec}_{\rm ERR} = \delta(\mathbf{x}) - \frac{2}{3}\delta^2_R(\mathbf{x}) - \mathbf{s}(\mathbf{x})\cdot\nabla\delta_R(\mathbf{x}) - \frac{1}{3}K^2_{RR}(\mathbf{x})$	$\delta^{\rm rec}_{\rm LRR} = \delta - \frac{1}{2}\left\{\delta_s[\mathbf{s}] + \delta_s[-\mathbf{s}]\right\}$

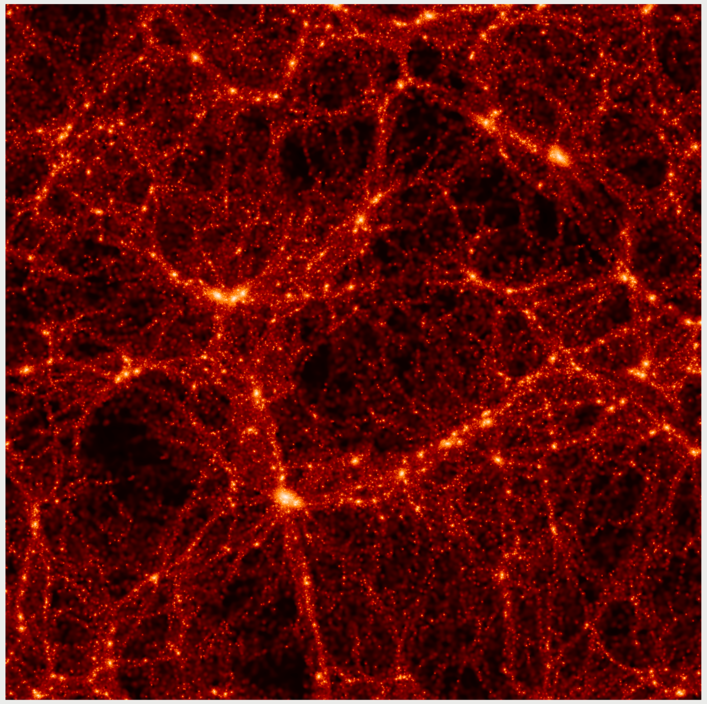
Equivalent in 2LPT*

* If displaced clustered catalogs are modelled such that displacement field is evaluated at Eulerian, not Lagrangian position, i.e. **s**[x] instead of **s**[**q**].

SIMULATIONS

SIMULATIONS

- FastPM code by Yu Feng
  - Particle-mesh code, time-stepping linear in scale factor a, parallel COLA
  - Ran wiggle and nowiggle simulations with same initial phases
  - 2048^3 DM particles, L=1380Mpc/h, 80 timesteps, 3 realizations, take 1% subsample at z=0.55; use R=15Mpc/h smoothing



- RunPB simulations by Martin White
  - Full TreePM (more accurate and expensive)
  - Only wiggle simulations

OVERVIEW

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- All reconstruction algorithms

	Eulerian	Lagrangian
Growth-Shift	$\delta^{\rm rec}_{\rm EGS}(\mathbf{x}) = \delta(\mathbf{x}) - \delta_R(\mathbf{x})\delta(\mathbf{x}) - \mathbf{s}(\mathbf{x})\cdot\nabla\delta(\mathbf{x})$	$\delta^{\rm rec}_{\rm LGS} = \delta_d[\mathbf{s}] - \delta_s[\mathbf{s}]$
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MS et al. 1508.06972

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NUMERICAL RESULTS: GROWTH-SHIFT ALGORITHMS

GROWTH-SHIFT ALGORITHMS

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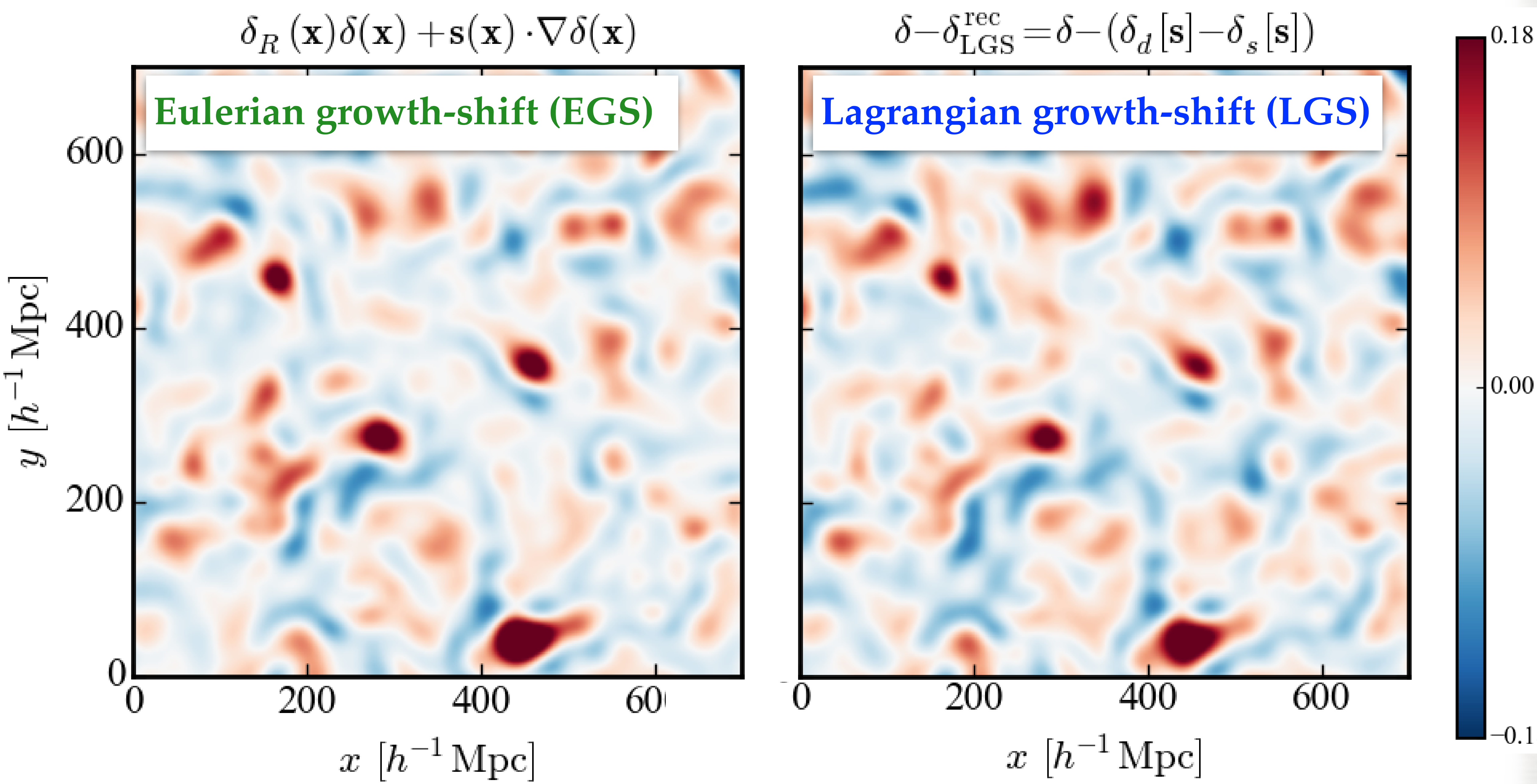
- **2D slice plots**

  Density change due to reconstruction, δ - δ_{rec}



GROWTH-SHIFT ALGORITHMS

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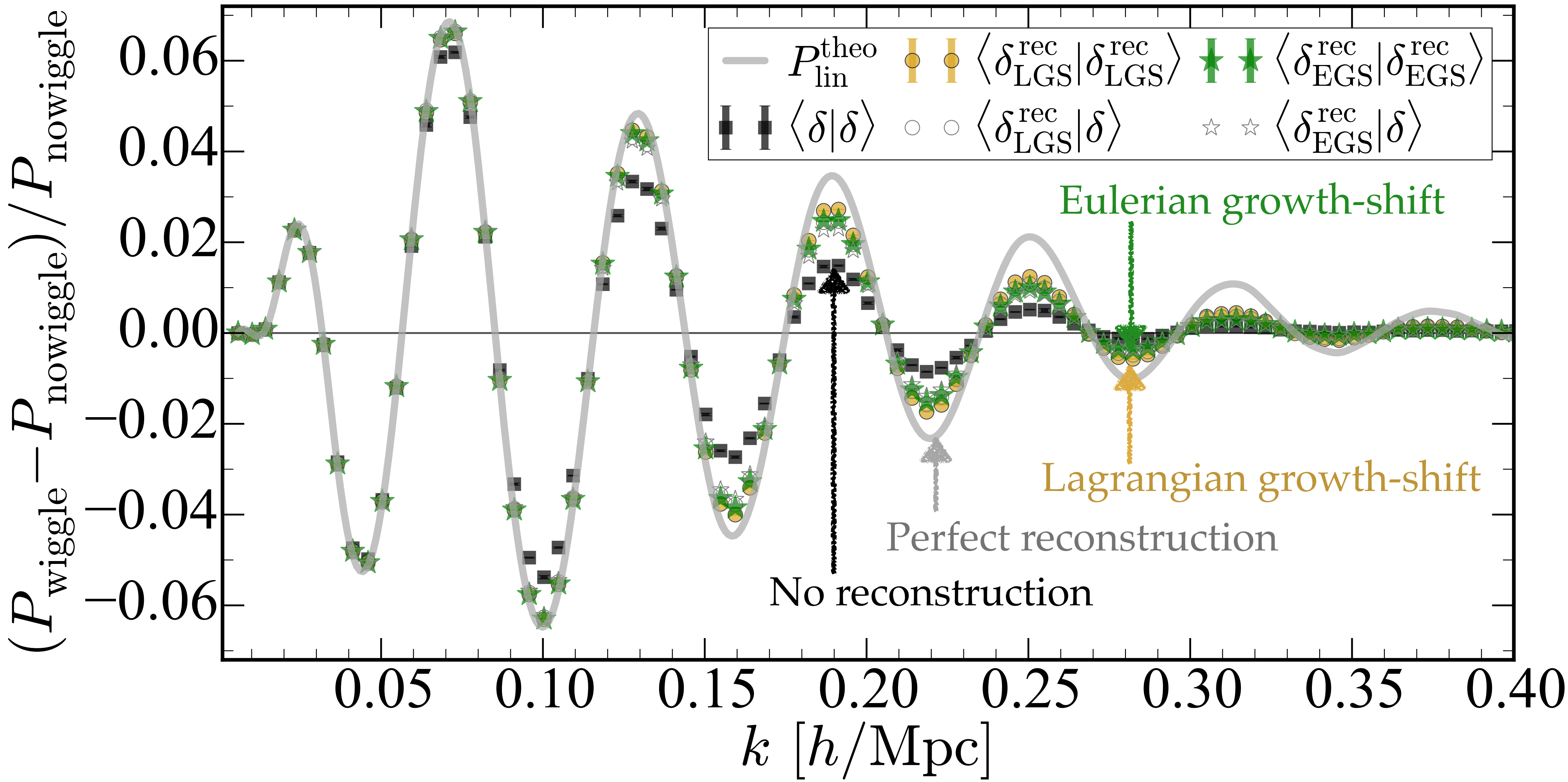
- **BAO signal**: Fractional difference of power spectra between wiggle and nowiggle simulations, for Lagrangian and Eulerian growth-shift algorithms



GROWTH-SHIFT ALGORITHMS

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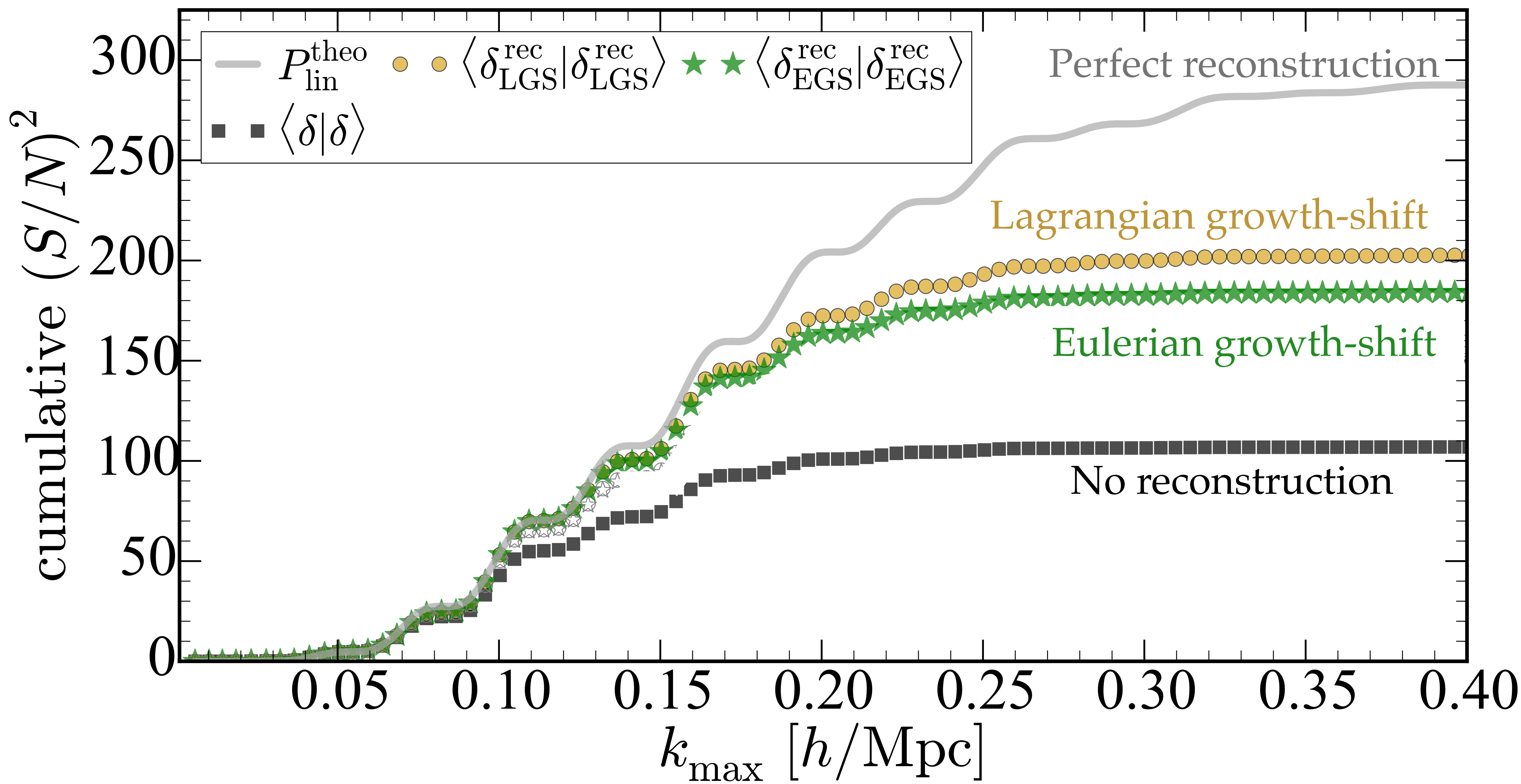
- **Cumulative BAO signal-to-noise-squared:**

  Eulerian algorithm yields 95% of BAO *S/N* of standard Lagrangian algorithm



WHERE DOES NEW BAO INFO COME FROM?

GROWTH-SHIFT ALGORITHMS

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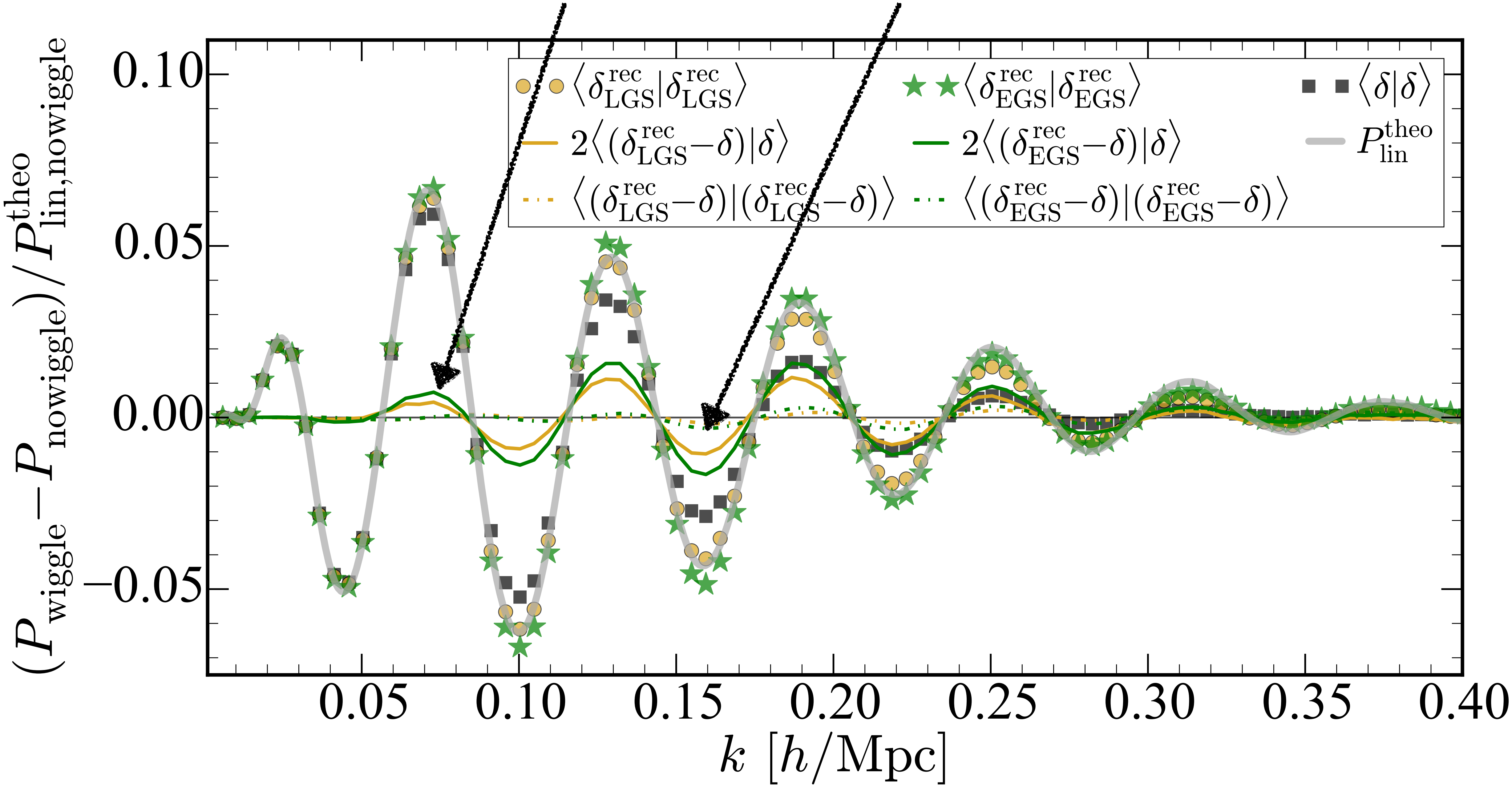
- **3-point vs 4-point:** Split $\delta_{\rm rec} = \delta + (\delta_{\rm rec}\text{-}\delta)$ in simulations

$$\Rightarrow \langle \delta^{\rm rec}|\delta^{\rm rec}\rangle = \underbrace{\langle\delta|\delta\rangle}_{\rm 2pt} + \underbrace{2\,\langle(\delta^{\rm rec}-\delta)|\delta\rangle}_{\rm 3pt} + \underbrace{\langle(\delta^{\rm rec}-\delta)|(\delta^{\rm rec}-\delta)\rangle}_{\rm 4pt}.$$



GROWTH-SHIFT ALGORITHMS

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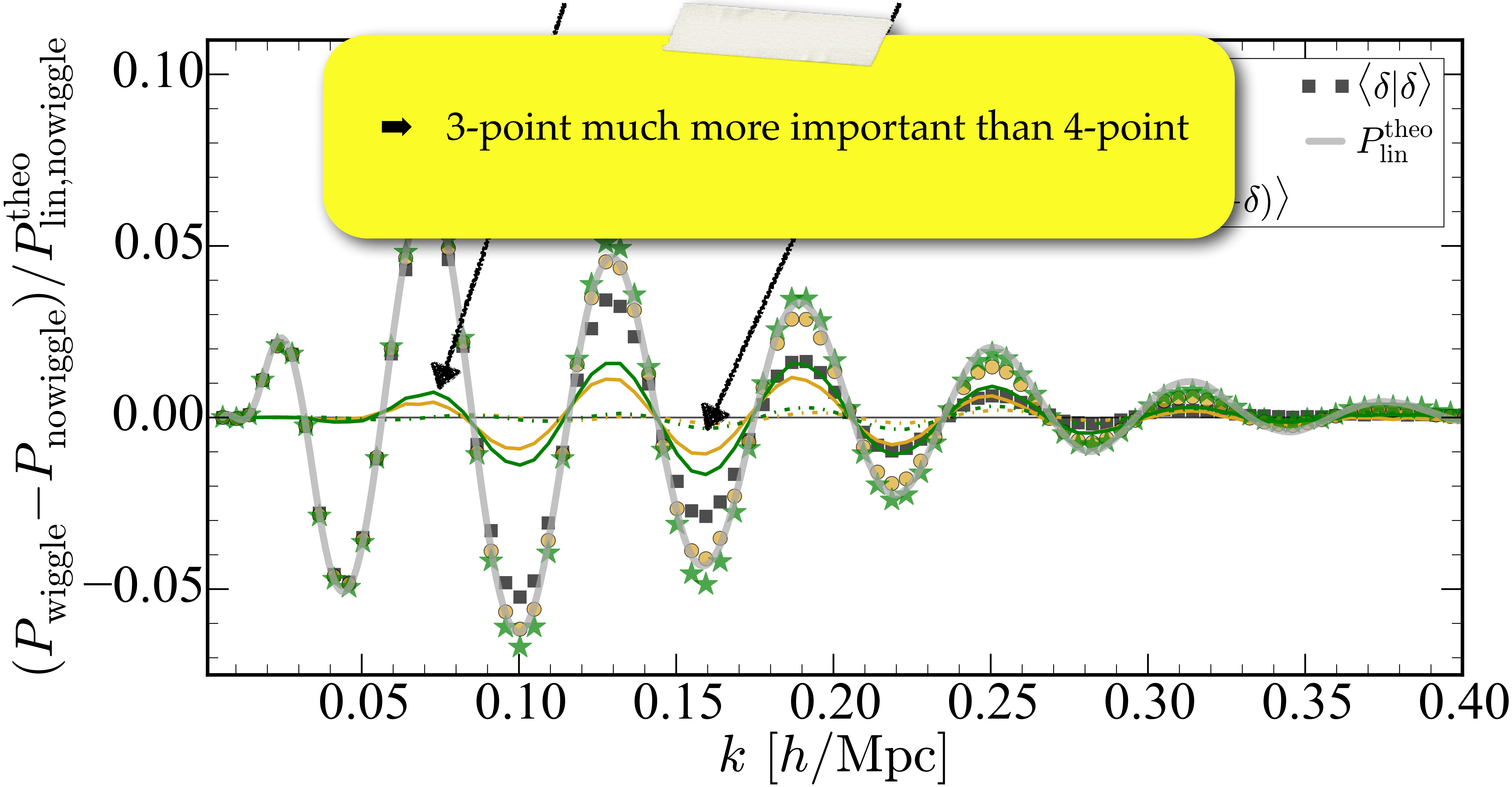

➡ 3-point much more important than 4-point

GROWTH-SHIFT ALGORITHMS

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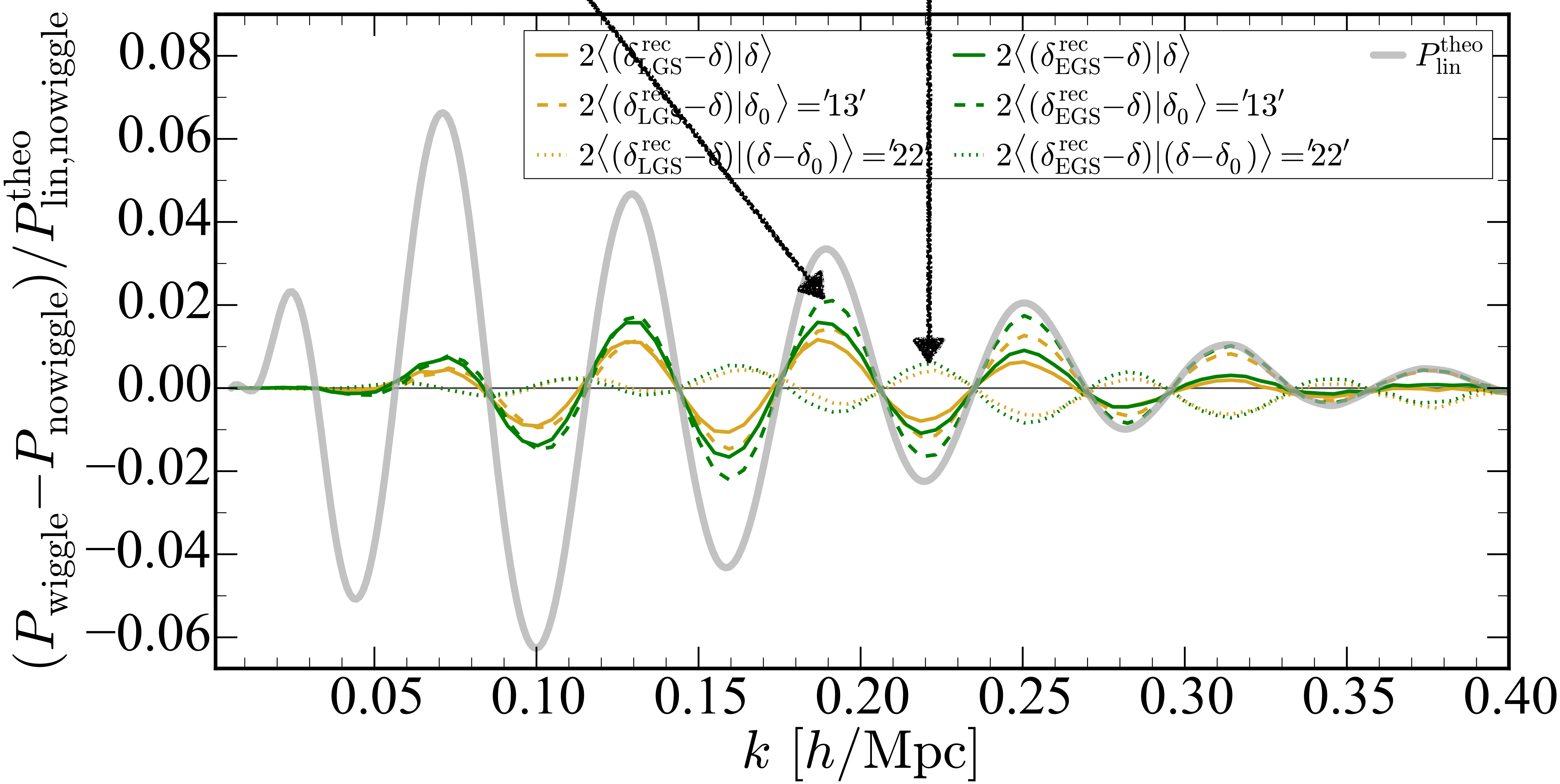
- **<u>13 vs 22 part (of 3-point):</u>** Split $\delta = \delta_0 + (\delta\text{-}\delta_0)$ in simulations, where δ_0 = linear density

$$\Rightarrow \langle(\delta^{\rm rec}-\delta)|\delta\rangle = \underbrace{\langle(\delta^{\rm rec}-\delta)|\delta_0\rangle}_{\sim\langle\Delta^{(3)}\delta_0\rangle} + \underbrace{\langle(\delta^{\rm rec}-\delta)|(\delta-\delta_0)\rangle}_{\sim\langle\Delta^{(2)}\delta^{(2)}\rangle}$$



GROWTH-SHIFT ALGORITHMS

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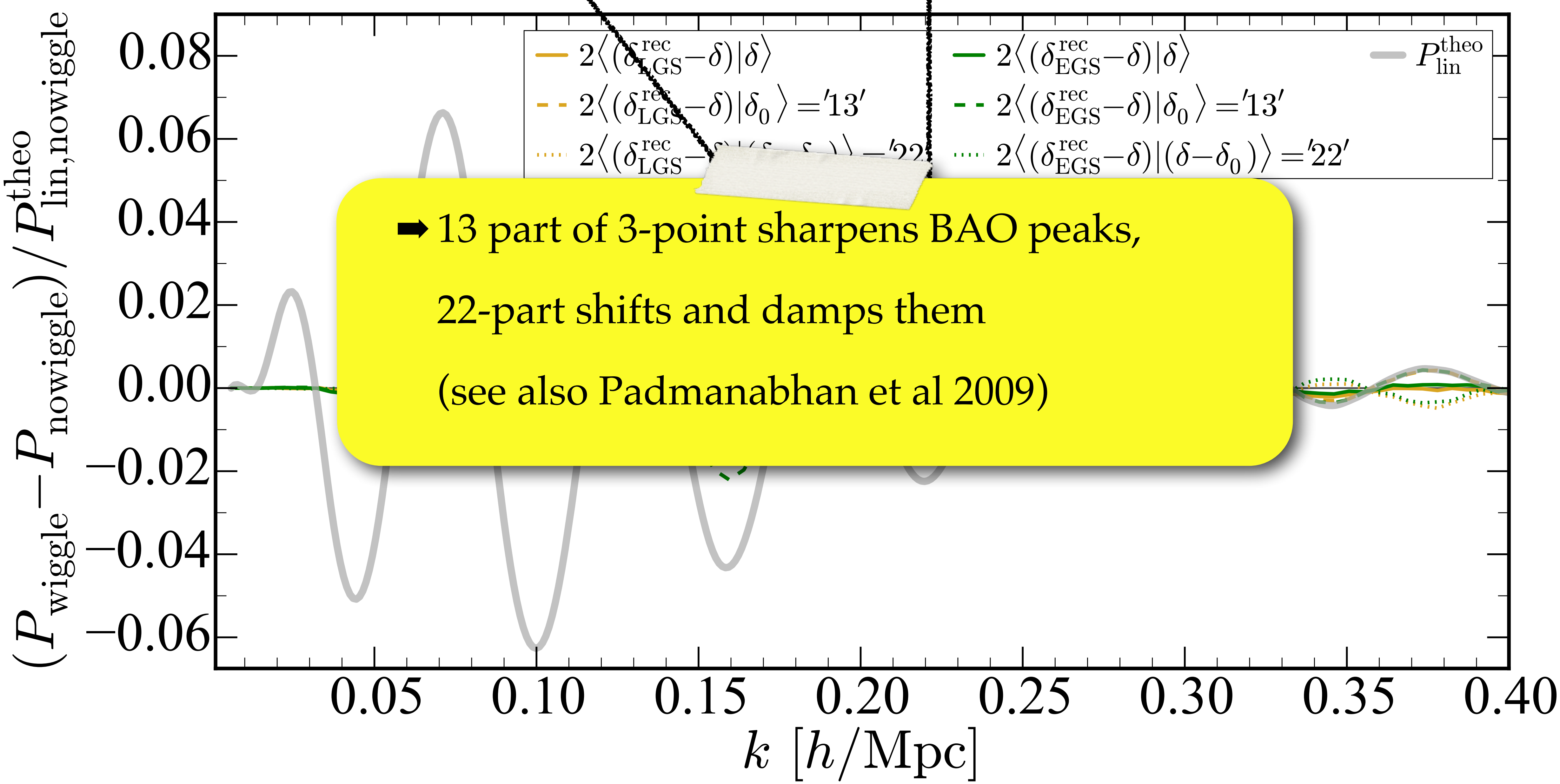

➡ 13 part of 3-point sharpens BAO peaks,

22-part shifts and damps them

(see also Padmanabhan et al 2009)

GROWTH-SHIFT ALGORITHMS

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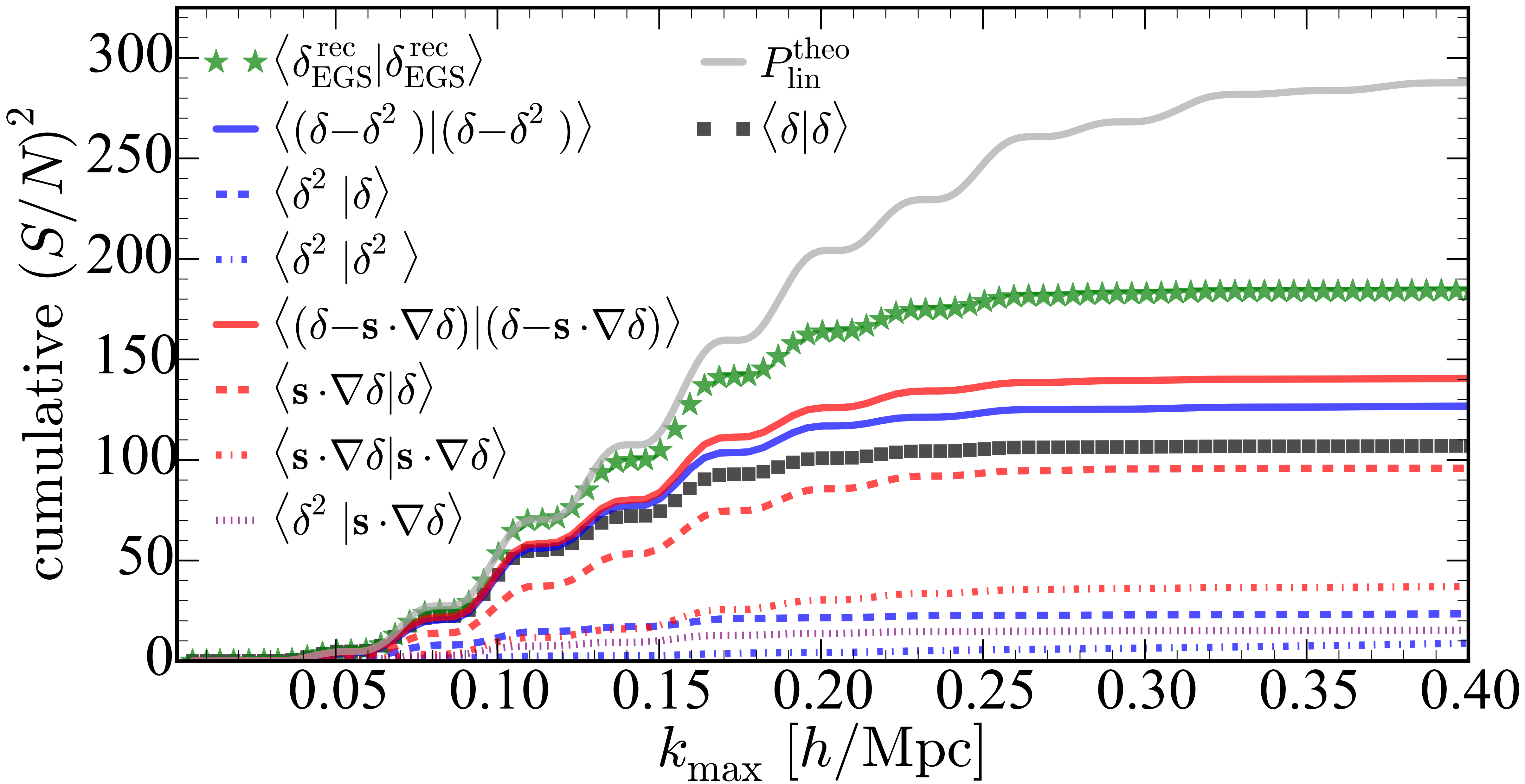
- **Growth vs shift part**



GROWTH-SHIFT ALGORITHMS

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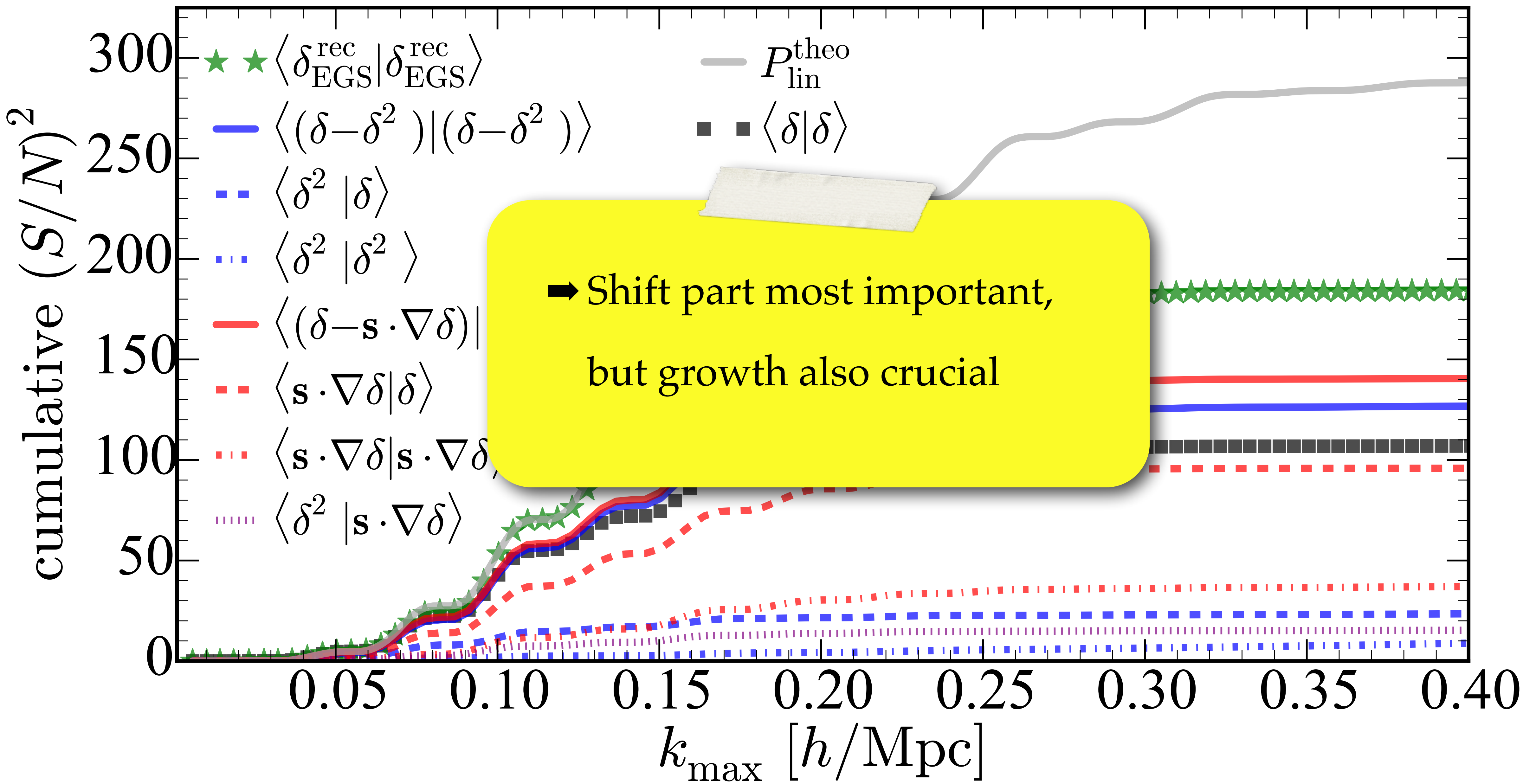

➡ Shift part most important, but growth also crucial

NUMERICAL RESULTS:

ALL

ALGORITHMS

ALL ALGORITHMS

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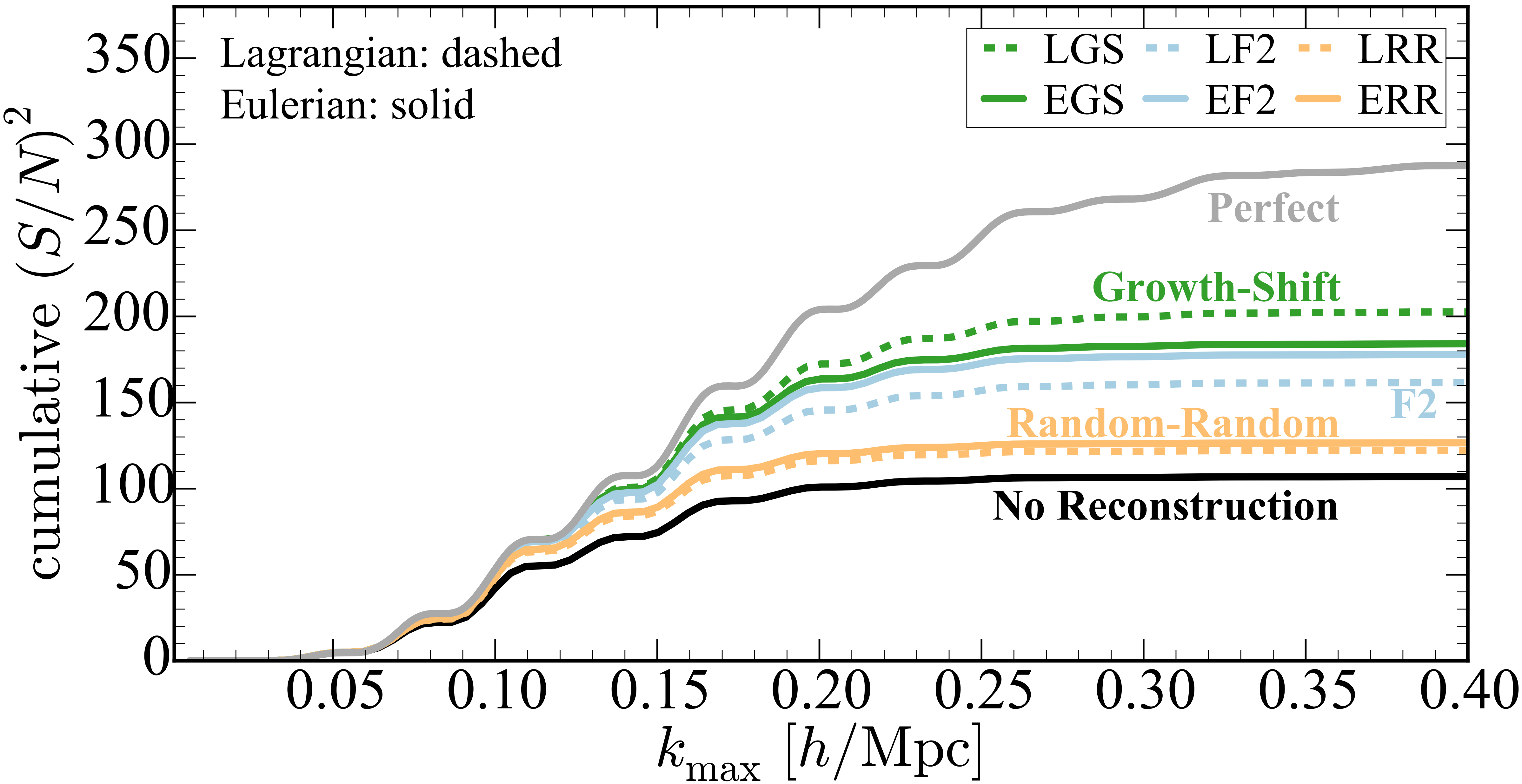
- **Performance comparison**
  - EGS algorithm yields 95% of BAO S/N of standard Lagrangian LGS algorithm
  - EF2 algorithm similar, other algorithms significantly worse



DISCUSSION

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- **Pros & Cons**

Eulerian reconstructions

+ BAO info comes from specific 3- & 4-point
+ Data transformations less dependent on fiducial model
+ Derived from nonperturbative continuity equation (EGS) or Newton-Raphson (EF2)
- Slightly worse performance
- Not worked out in redshift space

Lagrangian reconstructions

- Less transparent where BAO info comes from
- Data transformation very dependent on fiducial model
- Justification of algorithm mostly a posteriori

+ Slightly better performance
+ Well established (10 yr old), applied to observations

DISCUSSION

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- **Pros & Cons**

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Equivalent in 2LPT

➡ Reconstructions are connected

➡ New argument for success and robustness of standard reconstruction: implicitly includes 3- and 4-point

➡ Intuitively, expect little new BAO information in 3-point statistics, because rec. moves it into 2-point

CONCLUSIONS

CONCLUSIONS

MS, Baldauf, Seljak 1411.6595

- ***Simple bispectrum estimators***
  - Three cross-spectra of quadratic fields with the density measure projection of full bispectrum on tree-level shape
  - Simple, fast, nearly-optimal, simpler covariances
  - Works well in simulations
  - Extension to redshift space under development

MS et al. 1508.06972

- ***Eulerian BAO reconstruction and higher N-point statistics***
  - Presented 5 new BAO reconstruction algorithms
  - Showed connection to 3- & 4-point of unreconstructed density
  - Connected various algorithms to each other via 2LPT modelling
  - Standard algorithm performs best, but two Eulerian algorithms are almost as well, achieving ~95% BAO *S*/*N* of standard method

Thank you

BISPECTRUM BONUS SLIDES

THEORY CONTRIBUTIONS

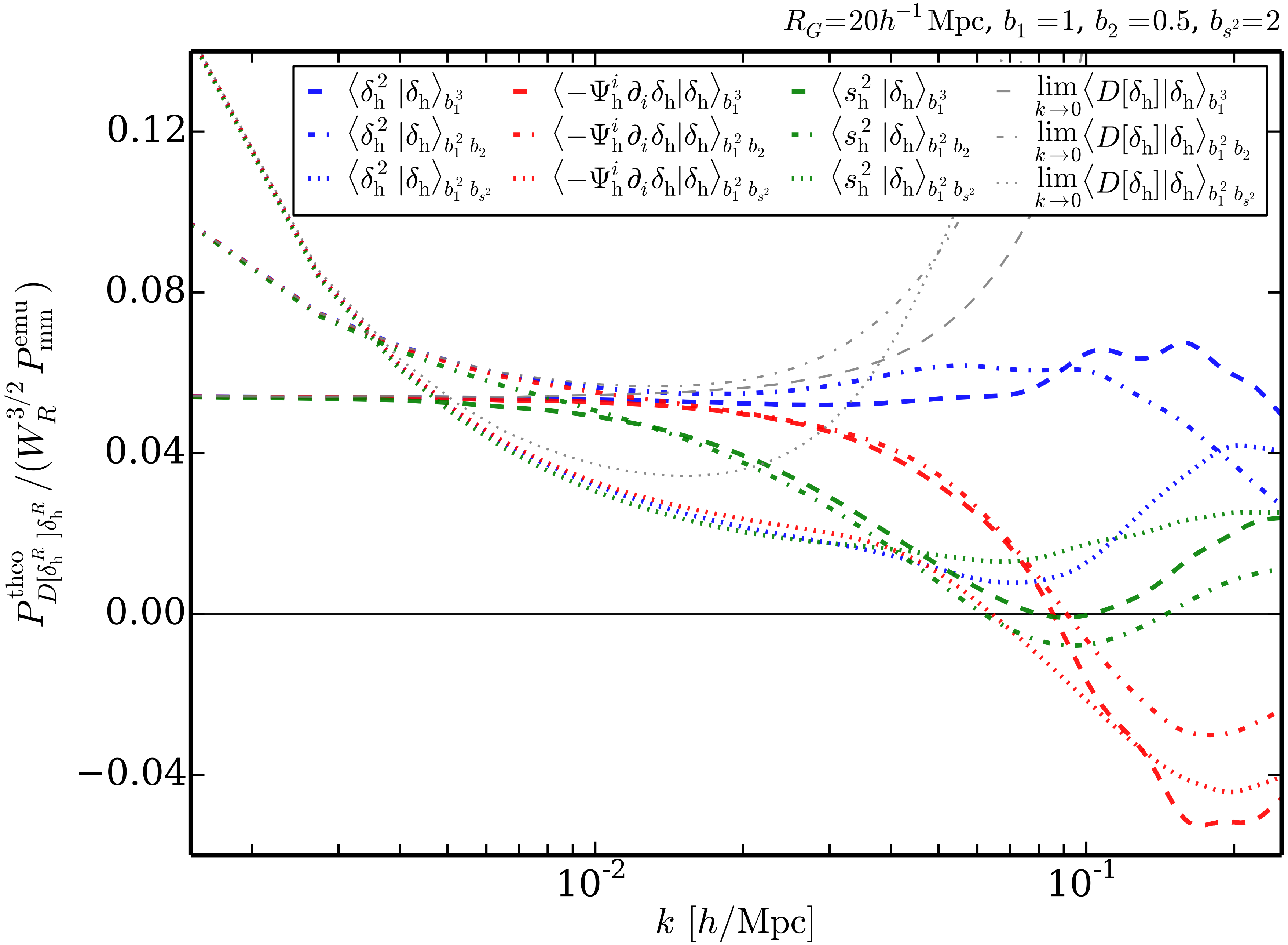

FIG. 1. Theory contributions (67) to halo-halo-halo cross-spectra scaling like b_1^3 (dashed), $b_1^2 b_2$ (dash-dotted) and $b_1^2 b_{s^2}$ (dotted) for squared density $\delta_{\rm h}^2(\mathbf{x})$ (blue), shift term $-\Psi^i_{\rm h}(\mathbf{x})\partial_i\delta_{\rm h}(\mathbf{x})$ (red) and tidal term $s^2_{\rm h}(\mathbf{x})$ (green), evaluated for fixed bias parameters $b_1 = 1$, $b_2 = 0.5$ and $b_{s^2} = 2$, Gaussian smoothing with $R_G = 20h^{-1}$Mpc, at $z = 0.55$, with linear matter power spectra in integrands. Thin gray lines show the large-scale (low k) limit given by Eq. (70). The cross-spectra are divided by the partially smoothed FrankenEmu emulator matter power spectrum $W_R^{3/2}P^{\rm emu}_{\rm mm}$ [45–48] for plotting convenience.

VERY LARGE SCALES

MS, Baldauf, Seljak, 1411.6595

- Large scale limit ($k \to 0$) of cross-spectrum expectation values:

$$\lim_{k\to 0} P_{D[\delta_h^R],\delta_h^R}(k) = W_R(k)\left[b_1^3 P_{\rm mm}^{\rm lin}(k)\left(\frac{68}{21}\sigma_R^2 - \frac{1}{3}\sigma_{R,P'}^2\right) + 2b_1^2 b_2\left(\tau_R^4 + 2P_{\rm mm}^{\rm lin}(k)\sigma_R^2\right) + \frac{4}{3}b_1^2 b_{s^2}\tau_R^4\right]$$

$$D \in \{\mathsf{P}_0, -F_2^1\mathsf{P}_1, \mathsf{P}_2\}$$

$$\sigma_R^2 \equiv \frac{1}{2\pi^2}\int \mathrm{d}q\, q^2 W_R^2(q) P_{\rm mm}(q)$$

$$\sigma_{R,P'}^2 \equiv \frac{1}{2\pi^2}\int \mathrm{d}q\, q^2 W_R^2(q) P_{\rm mm}(q)\frac{\mathrm{d}\ln q^3 P_{\rm mm}(q)}{\mathrm{d}\ln q}$$

$$\tau_R^4 \equiv \frac{1}{2\pi^2}\int \mathrm{d}q\, q^2 W_R^2(q) P_{\rm mm}^2(q)$$

- Different from position-dependent power spectrum (Chiang *et al.* 2014):

$$\lim_{k\to 0}\int \frac{d^2\Omega_{\hat{\mathbf{k}}}}{4\pi} B(\mathbf{k}-\mathbf{q}_1, -\mathbf{k}+\mathbf{q}_1+\mathbf{q}_3, -\mathbf{q}_3) = \left[\frac{68}{21} - \frac{1}{3}\frac{d\ln k^3 P(k)}{d\ln k}\right] P(k)P(q_3) + \mathcal{O}\left(\frac{q_{1,3}}{k}\right)^2$$

RECONSTRUCTION BONUS SLIDES

GROWTH-SHIFT ALGORITHMS

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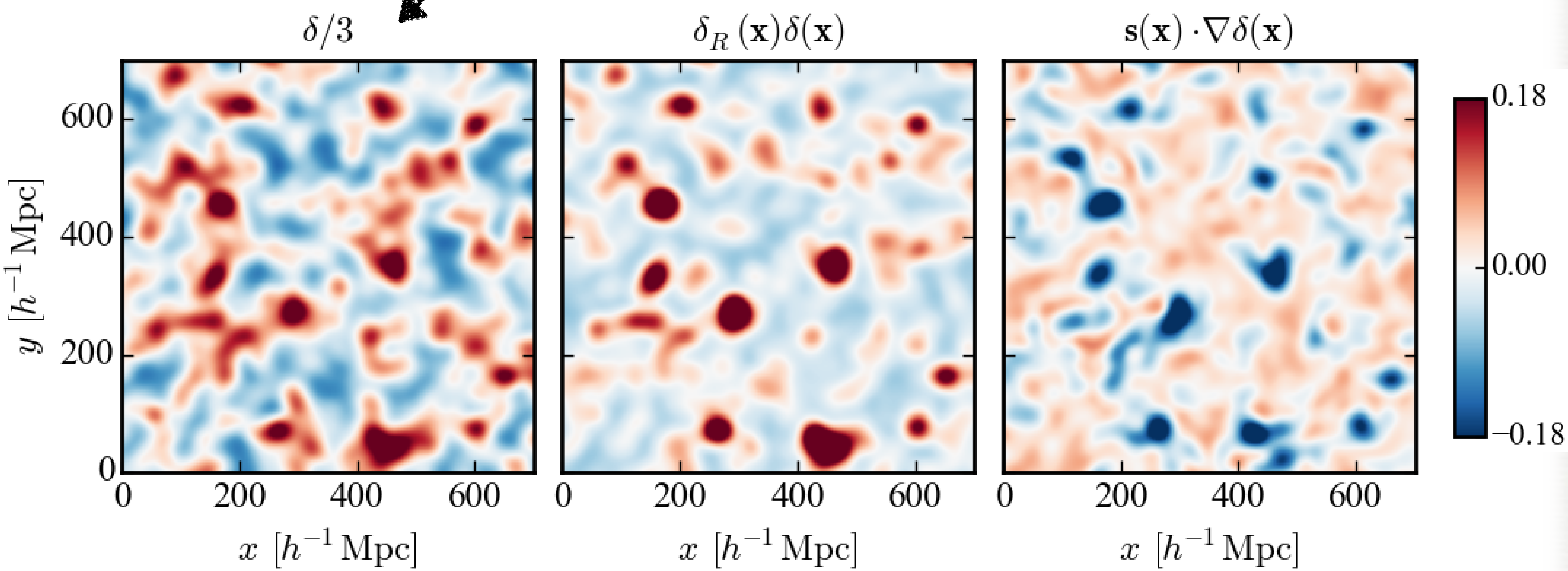
- **2D slice plots:** Components of EGS reconstruction

$$\delta^{\text{rec}}_{\text{EGS}}(\mathbf{x}) = \delta(\mathbf{x}) - \underbrace{\delta(\mathbf{x})\delta_R(\mathbf{x})}_{\text{growth}} - \underbrace{\mathbf{s}(\mathbf{x}) \cdot \nabla\delta(\mathbf{x})}_{\text{shift}}$$



ALL ALGORITHMS

MS et al. 1508.06972

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  - EGS algorithm yields 95% of BAO *S*/*N* of standard Lagrangian LGS algorithm
  - EF2 algorithm similar, other algorithms significantly worse

	Growth-Shift		F2 reconstruction		Random-Random			
Reconstruction method	LGS	EGS	LF2	EF2	LRR	ERR	Perfect	NoRec
BAO signal-to-noise	14.2	13.6	12.7	13.3	11.1	11.3	17.0	10.3
Compared against LGS	±0 %	−4.7 %	−11 %	−6.3 %	−22 %	−21 %	+19 %	−27 %
Compared against NoRec	+38 %	+31 %	+23 %	+29 %	+6.9 %	+8.8 %	+64 %	±0 %

TABLE II. Total BAO signal-to-noise for $k_{\mathrm{max}} = 0.4h/\mathrm{Mpc}$ for various reconstruction algorithms (obtained from Fig. 9, based on simulations). 'Perfect' refers to the BAO signal-to-noise of the linear density and 'NoRec' to the BAO signal-to-noise of the measured nonlinear density without performing any reconstruction. The second-to-last row shows how much of the signal-to-noise is lost compared to performing the standard LGS reconstruction. The bottom row shows how much signal-to-noise is gained by reconstructions compared to performing no reconstruction.

SMOOTHING SCALE

MS et al. 1508.06972

- **Dependence on smoothing scale**